# Solution Set: Homework 8

February 23, 2003

## 1 Integration Handout

### 1.1 Problem 19

Splitting the ice-ball into concentric shells, we have the volume of a shell $\Delta x$ thick of radius $x$ is:

$$4\pi x^2 \Delta x$$

The approximate amount of ice in this shell is:

$$4\pi x^2 \Delta x \rho(x)$$

Taking the Riemann Sum of all of the shells in the limite where $\Delta x \to 0$, we get the total amount of ice is:

$$\int_0^{10} 4\pi x^2 \rho(x)\, dx$$

### 1.2 Problem 20

1. The answer is the same and each of the steps are the same as above Problem 19. Cut the truffle into concentric shells, each of volume:

$$4\pi x^2 \Delta x$$

The approximate number of calories in this shell is:

$$4\pi x^2 \Delta x \rho(x)$$

Taking the Riemann Sum of all of the shells in the limite where $\Delta x \to 0$, we get the total number of calories is:

$$\int_0^R 4\pi x^2 \rho(x)\, dx$$

2. Here, the truffle must be cut into discs that have the same distance from the top of the shell. Call $x$ the distance from top of the truffle. The volume of the shell of thickness $\Delta x$ a distance $x$ from the top is:

$$\pi r(x)^2 \Delta x = \pi(R^2 - x^2)\Delta x$$

Here, $r(x)$ is the radius of the disc which is $\sqrt{R^2 - x^2}$ in this case. The approximate number of calories in this disc is $\delta(x)$ times this volume:

$$\pi(R^2 - x^2)\delta(x)\Delta x$$

And finally the limit of the Riemann sum yields the actual number of calories, by the integral:

$$\int_0^R 4\pi(R^2 - x^2)\delta(x)\,dx$$

### 1.3   Problem 21

1. The integral here is the same as the integral in Problem 20 (b) with $x := h$ and $R := 100$ and $\delta(x) := 6 \times 10^{-5}(200 - h)$ grams per cubic foot. The integral and answer is then:

$$\int_0^{100} 4\pi(100^2 - h^2)6 \times 10^{-5}(200 - h)\,dh$$

2. Evaluating this integral:

$$\int_0^{100} 4\pi(100^2 - h^2)6 \times 10^{-5}(200 - h)\,dh$$

$$= \quad 4\pi 6 \times 10^{-5} \int_0^{100} (10^4 - h^2)(200 - h)\,dh$$

$$= \quad 4\pi 6 \times 10^{-5}(\int_0^{100} 10^6 - h10^4 - h^2 200 + h^3\,dh$$

$$= \quad 6500\pi \approx 2.0 \times 10^4 \text{g}$$

## 2   Stewart, Section 6.2

### 2.1   Problem 28

The cross section should be a thin square. If we label $x$ starting from the center of the circle and going out, the volume of a square $\Delta x$ in thickness is:

$$V(x) = s(x)^2 = 4(r^2 - x^2)$$

Where $s(x)$ is the length of a side of the square which in this case is $2\sqrt{r^2 - x^2}$. The total volume is then the limit of the Riemman Sum:

$$V_{tot} = \int_{-r}^r V(x)\,dx = 8\int_0^r r^2 - x^2\,dx = 8[r^2 x - 1/3x^3]_0^r = 16/3r^3$$

## 2.2 Problem 37

1. $Volume(S_1) = \int_0^h A(z)dz = Volume(S_2)$ since the cross-sectional area $A(z)$ at height $z$ is the same for both solids.

2. By Cavalieri's Principle,m the volume of the cylinder in the figure is the same as that of a right circular cylinder of radius $r$ and height $h$ so the volume is $\pi r^2 h$.