Primer Assignment for Mathematics 1b
Problem Set # 0

This primer problem set is meant to prime you for the course you are about to begin. Students for whom the material is very fresh will be able to complete the assignment easily. The more review you need, the longer it will take. This is by design, in order to get everyone up to speed as the course begins. The problem set is due on the very first day of class.

Part I. Primer for Integration

The first third of this course deals with integration and its applications. You are expected to know the integrals in problem 1. If you don't, review section 5.3 in Stewart to learn them.

1. Evaluate the following:
   (a) $\int u^n\,du, \quad n \neq -1.$
   (b) $\int \frac{1}{u}\,du$
   (c) $\int \sin u\,du$
   (d) $\int \cos u\,du$
   (e) $\int \sec^2 u\,du$
   (f) $\int e^u\,du$
   (g) $\int b^u\,du$
   (h) $\int \frac{1}{1+u^2}\,du$
   (i) $\int \frac{1}{\sqrt{1-u^2}}\,du$

2. Knowing the integrals from problem 1, do the integrals below by substitution. (If you need review, look at section 5.5 in Stewart.)
   (a) i) $\int (2x+1)^3\,dx$ ii) $\int \frac{1}{(2x+1)^3}\,dx$ iii) $\int \frac{1}{2x+1}\,dx$
   (b) i) $\int x\sqrt{x^2+5}\,dx$ ii) $\int \sqrt{\cos x}\sin x\,dx$ iii) $\int t^2\sin(t^3)\,dt$ iv) $\int \tan t\,dt$
   (c) i) $\int_1^e \frac{\ln x}{x}\,dx$ ii) $\int \frac{e^{-x}}{e^x}\,dx$ iii) $\int \frac{x}{e^{x^2}}\,dx$

3. To prime yourself for applications, read Stewart section 5.2 and on page 438 from the Concept Check do: p. 438 # 2 and 5.

Part II. The Tangent line: the best linear approximation of a function at a point

Learning Goal: In the second unit of this course we will look at the 'best' quadratic approximation of a function at a point, the 'best' cubic approximation of a function at a point, etc. Looking at polynomial approximations of greater and greater degree will lead us to investigate 'infinite' polynomials, called infinite series. It is essential that you understand linear approximations thoroughly and that you have a picture in your mind of a linear approximation. This problem reviews linear approximations.

1. Use a linear approximation to estimate $\ln 1.1$. Is your approximation too big or too small? Why?

   Suggested approach:

   - *Sketch the graph of* $y = \ln x$.
     (We expect you to be able to do this at the drop of a hat. Can you graph $y = e^x$*? If so, then knowing the inverse relationship between* $\ln x$ *and* e^x *should be sufficient to graph* $\ln x$*. If you not sure you can graph* $y = e^x$ *then graph* $y = 3^x$*; the graphs look very similar.)*
   - *Off the top of your head, what's a good estimate for* $\ln 1.1$*? Give an integer value.*
   - *Is the number you picked above too large, or too small? The derivative (or the tangent line) can help you make an adjustment.*
   - *Sketch the graph of the line tangent to* $\ln x$ *at* $x = 1$*. The height of the point on this line corresponding to* $x = 1.1$ *is a good approximation to* $\ln 1.1$*.*

2. Use a linear approximation to estimate $\sqrt{24.8}$. Is your approximation too big or too small? Why?

Part III. Graphing Primer

Learning Goal: There are many different ways to look at mathematical problems; often a graphical approach is fruitful. In order to use this approach successfully, you need familiarilty with the graphs of some basic functions. For example, in addition to being able to graph lines and parabolas, you should have some expectations about what the graphs of higher order polynomials can look like. (This will be important for understanding Taylor and MacLaurin series.) You should be able to draw the graphs of trigonometric functions such as $\sin x$, $\cos x$ and $\tan x$, of exponential functions (such as e^x and e^{-x}), and of the logarithmic function. This is by no means an exhaustive list of all the functions you may run into in your studies, but it is a beginning. Some of you may have become accustomed to leaning very heavily on a graphing calculators for anything having to do with graphing. The exercises that follow are meant to prime your graphing skills. You should use a graphing calculator (or the graphing capacity of a computer) to *check* your work but not to do it.

1. How are the graphs of $y = f(x)$ and $y = f(x-2)$ related? If the zeros (roots) of $f(x)$ are at $x = 3, 7$, and 10, what are the zeros of $f(x-2)$?

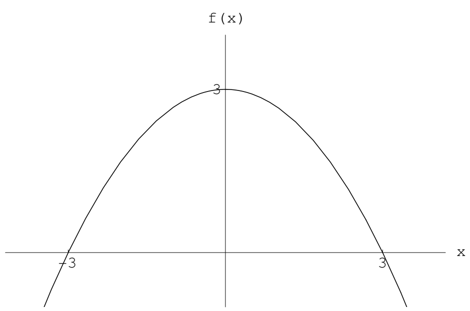
2. What are characteristics of polynomials that distinguish them from exponential, trigonometric, and logarithmic functions.

3. Look at the graphs below. Write a possible formula for each function. (There may be more than one correct answer to some of these problems.) Check your answer with a graphing calculator or a computer.

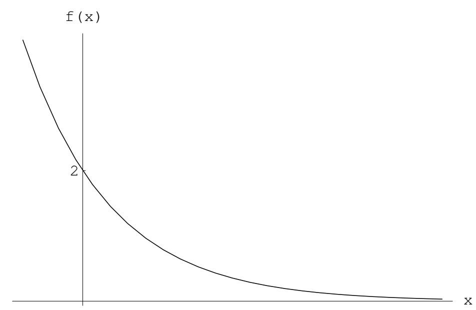
   The domain of the function in graph III is $\{x : x \geq -1\}$; all other functions have the domain $(-\infty, \infty)$. The functions graphed in IV and V are periodic. The function graphed in VII has infinitely many zeros and the zeroes are equally spaced.

   Hints: Two of the functions are basically trigonometric, two are basically exponential functions, one is a polynomial, one is a logarithmic funtion and one requires multiplication of different varieties of functions. The answers to some of these problems are not unique.

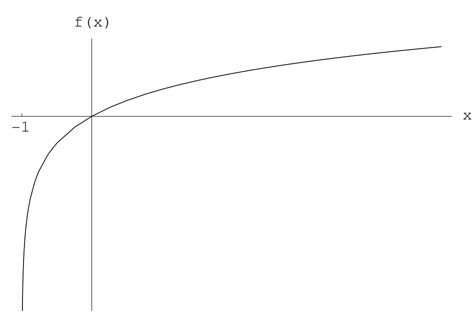
I.



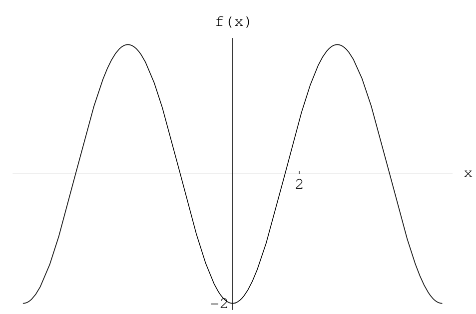
II.

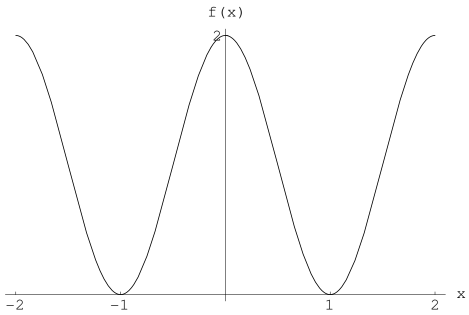


III.

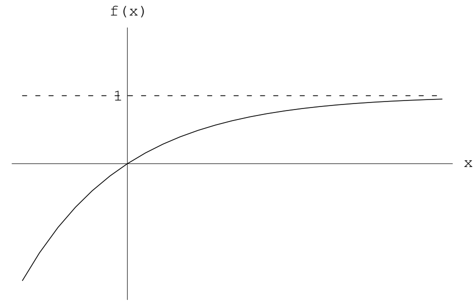


IV.

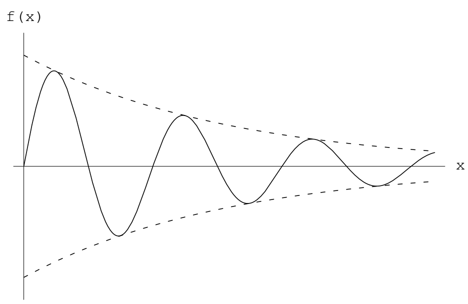




V.



VI.



VII.