Name: ______________________________ ID#: ____________________

Midterm I

Math 1b
Calculus, Series, Differential Equations

22 October 2003

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

This is a non-calculator exam.

Please circle your section:

MWF9	Matthew Leingang	TΘ10	Andrew Lobb
MWF10	Ken Chung	TΘ10	Chun-Chun Wu
MWF10	Florian Herzig	TΘ11:30	Amanda Alvine
MWF10	Michael Schein	TΘ11:30	Rosa Sena-Dias
MWF11	Matthew Leingang		
MWF11	Janet Chen		
MWF12	Ken Chung		

Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.

—Handbook for Students

Problem Number	Possible Points	Points Earned
1	12	
2	10	
3	20	
4	10	
5	15	
6	15	
7	18	
Total	100	

1. (12 Points) Compute the following limits, if they exist.

(a) $\lim_{n\to\infty} \frac{\ln(2+e^n)}{3n}$

(b) $\lim_{n\to\infty} \left(\frac{n+2}{n}\right)^{3n}$

(c) $\lim_{n\to\infty} \frac{\cos(n)}{\ln(n)}$.

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2. (10 Points) In January of 2003, Harvard Dining Services bought 100 eight-ounce cans of soup. In February, the factory decreased the amount of soup per can by 1% to 7.92 ounces, and HDS compensated by ordering 1% more cans (i.e., 101 cans). Assume that this pattern continues from month to month, that is, each month the can size decreases by 1% and the number of cans ordered increases by 1%[1], and that the soup is never eaten. How many ounces of soup are on hand after infinitely many months?

[1] Yes, this means that HDS starts ordering a non-integer number of cans eventually. We'll allow this in our theoretical universe.

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3. (20 Points) Determine whether the following series are convergent or divergent. In the case of a convergent series with negative terms in it, determine whether the convergence is absolute. Indicate clearly which tests you use and what conclusions you draw from them.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{(n+1)(n+2)}$

(c) $\sum_{n=1}^{\infty} 2^{1/n}$.

(d) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$.

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4. (10 Points) Let $A(x)$ be the *Airy function*

$$A(x) = 1 + \frac{x^3}{2 \cdot 3} + \frac{x^6}{2 \cdot 3 \cdot 5 \cdot 6} + \frac{x^9}{2 \cdot 3 \cdot 5 \cdot 6 \cdot 8 \cdot 9} + \cdots.$$

(a) If $A(x) = \sum_{n=0}^{\infty} c_n x^{3n}$, what is c_n?

(b) Find the interval of convergence of $A(x)$.

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5. (15 Points) Show that $(x^2 + x)e^x = \sum_{n=1}^{\infty} \frac{n^2 x^n}{n!}$ for all x.

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6. (15 Points) We will approximate $\dfrac{1}{\sqrt[3]{1.1}}$ with an error no bigger than 10^{-4}.

(a) Write the number as a series.

(b) Show that the terms in the series are alternating in sign.

(c) Assume that the other conditions for the Alternating Series Estimation Theorem are satisfied as well. Use it to estimate the sum of the series with the desired accuracy. (You can leave your answer as a fraction or a sum of fractions.)

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7. (18 Points) Label each of the following statments as true (**T**) or false (**F**). If the statement is true, explain why. If the statement is false, explain why or give an example that disproves the statement.

Do not assume more than is stated in the question. For instance, do not assume that sequences consist of all positive terms or that limits exist.

_____ (a) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges.

_____ (b) The series $\sum_{n=1}^{\infty} \sin(n)x^n$ converges if $|x| < 1$.

_____ (c) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} a_n^2$ is convergent.

______ (d) If $\{a_n\}$ is a positive sequence and $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} \ln(a_n)$ converges.

______ (e) If f is a continuous function and $a_n = f(n)$ and $\int_0^{\infty} f(x)\, dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

______ (f) If $\sum_{n=1}^{\infty} c_n x^n$ converges at $x = 1$, then $\sum_{n=1}^{\infty} n c_n x^{n-1}$ converges at $x = 1$.

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