Math 1a. Lecture 1
Course Introduction

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1 Goals

- Course introduction and getting to know the students.

- Reviewing the definite integral and the Fundamental Theorem of Calculus.

2 Reviewing The Definite Integral

The rate at which the world's oil is being consumed is increasing. Suppose that the rate (measured in billions of barrels per year) is given by the function $r(t)$, where t is measured in years and $t = 0$ on January 1, 1990.

1. Write a definite integral that represents the total quantity of oil used between the start of 1990 and the start of 1995.

2. Suppose that $r(t) = 32e^{0.05t}$. Find the approximate value for the definite integral from part (a) using a right-hand sum with $n = 5$ subintervals.

3. Interpret each of the five terms in the sum from part (b) in terms of oil consumption.

4. Evaluate the definite integral from part (a) exactly using the Fundamental Theorem of Calculus.

3   Math 1a Final Exam Problems

- Which of the following are equal to

$$\int_1^5 \frac{\ln x}{x}\, dx?$$

Please circle all of the correct answers. You do not need to justify your solution.

(a) $\displaystyle\sum_{i=1}^{5} \frac{\ln x_i}{x_i}\Delta x$, where $\Delta x = 4/n$ and $x_i = 1 + i\Delta x_i$.

(b) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{5} \frac{\ln x_i}{x_i}\Delta x$, where $\Delta x = 4/n$ and $x_i = 1 + i\Delta x_i$.

(c) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \frac{\ln x_i}{x_i}\Delta x$, where $\Delta x = 4/n$ and $x_i = 1 + i\Delta x_i$.

(d) $(\ln 5)^2/2$

(e) $\dfrac{1}{2}\left(\dfrac{\ln 5}{5}\right)^2 - \dfrac{1}{2}(\ln 1)^2$

(f) $\dfrac{1}{5^2} - \dfrac{1}{1^2}$

(g) $\ln(\ln(5)) - \ln(\ln 1))$

(h) $\dfrac{\ln 1}{1} + \dfrac{\ln 2}{2} + \dfrac{\ln 3}{3} + \dfrac{\ln 4}{4}$

- Put the following in *ascending* order (with "=" or "<" signs between each expression. You do not need to justify your solution. [*Hint*: Think about which expressions are positive, which are negative, and which are zero. A picture may be helpful.]

(a) $\displaystyle\int_2^6 \ln t\, dt$

(b) $\ln 2 + \ln 3 + \ln 4 + \ln 5$

(c) $\ln 3 + \ln 4 + \ln 5 + \ln 6$

(d) zero

(e) $\ln(2/6)$

(f) $\displaystyle\lim_{h\to 0}\frac{\ln(2+h) - \ln 2}{h}$

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4 Fundamental Theorem of Calculus

Part I. If f is a continuous function on $[a, b]$, then the function

$$g(x) = \int_a^x f(t)\, dt$$

is defined for all $a \leq x \leq b$. Furthermore, g is continuous on $[a, b]$ and differentiable on (a, b) with

$$g'(x) = \frac{d}{dx} \int_a^x f(t)\, dt = f(x).$$

Part II. If F is any antiderivative of f, then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

References

- §5.1–5.4 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0–534–41022–7.

Notes

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