Math 1a. Lecture 2
Integration by Substitution

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1 Goals

- To be able to evaluate definite and indefinite integrals using substitution.

2 Integration by Substitution

The general integration by substitution formula is

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\,du$$

where $u = g(x)$ and $du = g'(x)\,dx$.

3 Substitution for Definite Integrals

If $u = g(x)$ and $du = g'(x)\,dx$,

$$\int_a^b f(g(x)) \cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

Choose an antiderivative F or f. Then $F' = f$ and the Chain Rule implies

$$\frac{d}{dx}F(g(x)) = F'(g(x)) \cdot g'(x) = f(g(x)) \cdot g(x).$$

Therefore,

$$\begin{aligned}\int f(g(x)) \cdot g'(x)\,dx &= F(g(x))\Big]_a^b \\ &= F(g(b)) - F(g(a)) \\ &= F(u)\Big]_{g(a)}^{g(b)} \\ &= \int_{g(a)}^{g(b)} f(u)\,du.\end{aligned}$$

4 Taking Advantage of Symmetry

Suppose that f is continuous on $[-a, a]$. Then

- If f is even, then
$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx.$$
- If f is odd, then
$$\int_{-a}^{a} f(x)\,dx = 0.$$

5 Key Examples

- $\int (4x+3)^3\,dx$
- $\int_0^{\pi} 4\sin^3 x \cos x\,dx$
- $\int x^3\sqrt{9-x^2}\,dx$
- $\int \tan x\,dx$
- Suppose that $\int_0^{12} g(x)\,dx = \pi$. Evaluate $\int_0^4 g(3x)\,dx$.

References

- §5.6 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0–534–41022–7.

Notes

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