Math 1a. Lecture 3
Integration by Parts

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1 Goals

- To be able to evaluate definite and indefinite integrals using integration by parts.

2 Integration by Parts

- $\int u \, dv = uv - \int v \, du$

- $\int_a^b u \, dv = uv \Big]_a^b - \int_a^b v \, du$

Since

$$(u \cdot v)' = u \cdot v' + u' \cdot v.$$

we can integrate both sides to obtain

$$\int u \, dv = uv - \int v \, du.$$

3 Key Examples

- $\int xe^x \, dx$ and $\int_0^1 xe^x \, dx$

- $\int x^2 e^x \, dx$

- $\int \ln x \, dx$

- $\int e^x \cos x \, dx$

4 LIATE

Students often ask how should I choose u and dv. The answer is *LIATE*. That is, choose u in the following order:

- **L**ogarithmic

- **I**nverse

- **A**lgebraic

- **T**rigonometric

- **E**xponential

For example, let $u = \arctan x$ and $dv = dx$ in $\int \arctan x \, dx$, since $\arctan x$ is an **I**nverse function. Then

$$
\begin{aligned}
\int \arctan x \, dx &= uv - \int v \, du \\
&= x \arctan x - \int \frac{x}{1 + x^2} \, dx \\
&= x \arctan x - \frac{1}{2} \ln(1 + x^2).
\end{aligned}
$$

References

- §5.6 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0–534–41022–7.

Notes

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