Math 1a. Lecture 5
Additional Techniques of Integration
(Partial Fractions)

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Spring 2006

1   Goals

- To be able to evaluate definite and indefinite integrals using partial fractions.

2   Integration by Partial Fractions

Rational functions can be written as the sum of functions of the form

- $\dfrac{A}{(ax+b)^n}$

- $\dfrac{Ax+B}{(ax^2+bx+c)^n}$

For example,

$$\int \frac{2}{1-x^2}\,dx = \int \frac{dx}{1+x} + \frac{dx}{1-x} = \ln|1+x| - \ln|1-x| + C.$$

This is called a *partial fraction decomposition*.

3   Key Examples

- $\displaystyle\int \frac{x^3}{1+x^2}\,dx$

- $\displaystyle\int \frac{2+5x+3x^2+3x^3}{x(1+x^2)(x+2)}\,dx = \int \frac{dx}{x} + \int \frac{dx}{1+x^2} + \int \frac{2\,dx}{x+2}$

Show students how to find the partial fraction decomposition.

4   Notes on the Worksheet

1. $\displaystyle\int \frac{5x+7}{(x+1)(x+2)}\, dx$

2. $\displaystyle\int \frac{x^2+3x-1}{x(x+1)(x-2)}\, dx$

3. $\displaystyle\int \frac{x^2-1}{x(x^2+4)}\, dx$

4. $\displaystyle\int \frac{3x^2+7x+5}{(x+1)(x^2+2x+2)}\, dx$

5. $\displaystyle\int \frac{4x^2-3x+2}{x(2x-1)^2}\, dx$

   Hint: Let $\displaystyle \frac{4x^2-3x+2}{x(2x-1)^2} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$.

6. $\displaystyle\int \frac{1}{x^2+2x+2}\, dx = \int \frac{1}{(x^2+2x+1)+1}\, dx$

References

- §5.7 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0–534–41022–7.

Notes

February 15, 2006