Math 1a. Lecture 8
More about Areas

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Spring 2006

1 Goals

- To be able to calculate the area between two curves and to understand the connection between

$$\int_a^b f(x) - g(x)\, dx$$

and Riemann Sums.

2 Finding the Area between Two Curves

Suppose that we wish to find the area between $\sin x$ and $\cos x$ on the interval $[\pi/4, 5\pi/4]$.

Let $f(x)$ and $g(x)$ be defined on an interval $[a, b]$ with $f(x) \geq g(x)$ for all $x \in [a, b]$. If we wish to approximate the area between the two curves, we can partition the interval as

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

and estimate the area as a Riemann Sum,

$$\text{Area} = \sum_{i=1}^{n} A_i \approx \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]\Delta x,$$

where $\Delta x = x_i - x_{i-1}$ and $x_i^* \in [x_{i-1}, x_i]$.

Thus, we can define the area between two curves to be a Riemann integral

$$\text{Area} = \int_a^b [f(x) - g(x)]\, dx.$$

Thus, the area between $\sin x$ and $\cos x$ on the interval $[\pi/4, 5\pi/4]$ is

$$\int_{\pi/4}^{5\pi/4} \sin x - \cos x \, dx = -\cos x - \sin x \Big|_{\pi/4}^{5\pi/4} = 2\sqrt{2}.$$

3 Worksheet

1. Find the area bounded by

$$\begin{aligned} x &= 0 \\ x &= 2 \\ y &= \frac{1}{x+1} \\ y &= \sqrt{x+2} \end{aligned}$$

2. Find the area bounded by

$$\begin{aligned} y &= x \\ y &= x^2 \end{aligned}$$

3. Find the area bounded by

$$\begin{aligned} x &= y^2 - 4y \\ x &= 2y - y^2 \end{aligned}$$

4. A cross-section of an airplane wing has length 200 cm. Measurements of the thickness of the wing, in centimeters, at 20-centimeter intervals are 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, and 2.8. Use Simpson's Rule to estimate the are of the wing's cross-section.

5. Find the area bounded by

$$\begin{aligned} y &= |x| \\ y &= x^2 - 2 \end{aligned}$$

6. Find the area bounded by

$$\begin{aligned} y &= 1/x \\ y &= x \\ y &= x/4 \end{aligned}$$

References

- §6.1 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0–534–41022–7.

Notes

February 27, 2006