

# Math 1a. Lecture 9

## Volumes

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### 1 Goals

- To understand and be able to calculate volumes by integrating over cross-sectional area.
- To understand and be able to calculate volumes of solids of revolution using<sup>1</sup>

$$V = \int_a^b \pi[f(x)]^2 dx.$$

### 2 Some Volumes that We Know

- Rectangular Box:  $V = lwh$
- Cylinder:  $V = \pi r^2 h$
- Cone:  $V = \frac{1}{3}\pi r^2 h$
- Sphere:  $V = \frac{4}{3}\pi r^3$

### 3 Definition of Volume

We can approximate the volume of a solid if we know the area of a cross-section at each point. If  $S$  is a solid lying between  $a$  and  $b$  and  $A(x)$  is the

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<sup>1</sup>We will not cover the method of cylindrical shells.

cross-sectional area of  $S$  on a the plane passing through  $x$  and perpendicular to the  $x$ -axis, then

$$V = \sum_{i=1}^n V_i \approx \sum_{i=1}^n A(x_i^*) \Delta x_i.$$

We define volume to be

$$V = \sum_{i=1}^n V_i = \lim_{n \rightarrow \infty} \sum_{i=1}^n A(x_i^*) \Delta x_i = \int_a^b f(x) dx.$$

## 4 The Volume of a Sphere

Show that the volume of a sphere of radius  $r$  by rotating  $y = \sqrt{r^2 - x^2}$  about the  $x$ -axis is  $(4/3)\pi r^3$ . Note that this leads to finding the volumes of solids of revolution

$$V = \int_a^b \pi [f(x)]^2 dx.$$

where  $f$  is rotated about the  $x$ -axis on the interval  $a \leq x \leq b$ .

## 5 Worksheet Problems

1. Find the volume of a sphere of radius  $r$  by rotating  $y = \sqrt{r^2 - x^2}$  about the  $x$ -axis.
2. Find the volume obtained by rotating the region bounded by  $y = x^3$  and  $y = x$  for  $x \geq 0$  about the  $x$ -axis.
3. Find the volume obtained by rotating the region bounded by  $y = \sqrt{x}$  and  $y = x$  about the line  $y = 1$ .
4. The base of a solid object  $S$  is the region bounded by the parabola  $y^2 = 3x$  and the line  $x = 3$ . Cross-sections for the object perpendicular to the  $y$ -axis are equilateral triangles. What is the volume of  $S$ ?

## References

- §6.2 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0-534-41022-7.

## Notes

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