Math 1a. Lecture 10
Applying the Definite Integral: Slice and Conquer

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1 Goals

- To understand and be able to use Riemann Sums and the definite integral to calculate "mass" when density varies.

2 Worksheet Examples

1. A city is in the shape of a rectangle 4 miles wide and 6 miles long. A river runs through the middle of the city, parallel to the 6-mile long sides. People prefer to live nearer the water, so the density of people is given by $\rho(x) = 10000 - 800x$ people per square mile, where x is the distance from the river.

   (a) Show in a sketch how to slice up the region.

   (b) What is the area of the ith slice?

   (c) What is the approximate population of the ith slice?

   (d) Write a Riemann sum to estimate the total population of the city.

   (e) Calculate the population of the city using a definite integral.

2. A village is in the shape of a circle of radius 3 kilometers. A well, located in the center of the city, is the only source of water. People prefer to live nearer the well, so the density of people is given by $\rho(x) = 800 - 100x$ people per square mile, where x is the distance from the well.

   (a) Show in a sketch how to slice up the region.

   (b) What is the area of the ith slice?

(c) What is the approximate population of the ith slice?

(d) Write a Riemann sum to estimate the total population of the city.

(e) Calculate the population of the city using a definite integral.

3. A beam of light is shining onto a screen creating a disk of radius 50 cm. The intensity of the light is brightest at the center and diminishes away from the center. If the intensity of the light at a distance r from the center of the bean is given by

$$f(r) = \frac{150}{20 + r^2}$$

watts per cm^2, find the total wattage of the beam's image on the screen.

4. Suppose the density of a mass in a gaseous planet is given by the function

$$\rho(r) = \frac{40000}{1 + 0.0001r^3}$$

kilograms per cubic kilometer, where r is the distance in kilometers from the center of the planet. Find the total mass of the planet if its radius is 8000 km.

References

- §27.1 in Robin Gottlieb. *Calculus: An Integrated Approach to Functions and their Rates of Change*. Addison Wesley, Boston, 2002. ISBN 0–201–70929–5.

Notes

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