Math 1a. Lecture 12
Applications to Physics and Engineering (Work)

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1 Goals

- To understand and be able to apply the definite integral when computing work done.

2 Work

- If an object of mass m moves along a straight line with position function $s(t)$, then the force on the object (in the same direction) is given by Newton's Second Law of Motion

$$F = m\frac{d^2s}{dt^2}.$$

  Force is measured in Newton's (N = kg·m/s^2). A force of one newton on a mass of one kilogram produces an acceleration of 1 m/s^2.

- For constant acceleration, force is constant and work is given by

$$\text{Work} = \text{Force} \times \text{Distance}.$$

  We measure work in newton-meters or *joules*. If force is measured in pounds and distance in feet, then we measure work in foot-pounds,

$$1 \text{ ft-lb} \approx 1.36 \text{ J}.$$

- The work done lifting a 30 lb toddler to hip height (3 ft) is

$$\begin{aligned} 30 \text{ lb} \times 3 \text{ ft} &= 90 \text{ ft-lb} \\ 133.44 \text{ N} \times 0.914 \text{ m} &= 121.964 \text{ J} \end{aligned}$$

- Suppose a continuous force $f(x)$ acts on an object over the interval $a \le x \le b$. The work done on a subinterval is

$$W_i \approx f(x_i^*)\Delta x_i,$$

and the total work done can be approximated by

$$W \approx \sum_{i=1}^{n} f(x_i^*)\Delta x_i.$$

We now define work done by moving an object from $x = a$ to b by

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x_i = \int_a^b f(x)\,dx.$$

3 Examples

1. *Hooke's Law* tells us that the force created by a spring when it is stretched or compressed is proportional to the distance the spring is stretched or compressed,

$$F(x) = kx.$$

   If a spring stretched 1 foot beyond its natural length requires 10 lbs of force, how much work is done stretching the spring?

2. Is it more work to raise a 70 lb bucket from a 50 ft well or a 50 lb bucket from a 70 ft well if the rope weighs 0.25 lb per foot?

3. A circular swimming pool has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. If the density of water is 62.5 lb/ft^3, how much work is required to pump all of the water out over the side?

References

- §6.5 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0–534–41022–7.

Notes

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