Math 1b. Lecture 14
Series—Geometric Series

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1 Goals

- To understand and be able to apply geometric series.

2 An Example

Suppose that the country of Pottsylvania spends \$2 billion and that each recipient of a fraction of this wealth spends 90% of the dollars that he or she receives. In turn, the secondary recipients spend 90% of the dollars that they receive, and so on. What is the total spending that results from the original injection of \$2 billion dollars into the economy?

$$2 + 0.9 \cdot 2 + (0.9)^2 \cdot 2 + (0.9)^3 \cdot 2 + \cdots$$

3 Geometric Series

Let

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + \cdots .$$

The nth partial sum is

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^n = a\frac{1 - r^{n+1}}{1 - r}.$$

- If $|r| < 1$, then $\displaystyle\sum_{k=0}^{\infty} ar^k = \frac{a}{1 - r}$.
- If $|r| \geq 1$, the series diverges.

4 Worksheet Problems

1. Suppose that the country of Pottsylvania spends \$2 billion and that each recipient of a fraction of this wealth spends 90% of the dollars that he or she receives. In turn, the secondary recipients spend 90% of the dollars that they receive, and so on. What is the total spending that results from the original injection of \$2 billion dollars into the economy?

2. Determine the convergence of each of the following geometric series. If the series converges, find its sum.

   (a) $1 + 0.4 + 0.16 + 0.064 + \cdots$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-6)^n}{5^n}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{3^n}{\pi^n}$

3. Express $6.254254\ldots$ as a ratio of two integers.

4. A ball is dropped from a height of 6 ft. Each time the ball bounces, it comes back up to one-half of its previous height. What is the total distance that the ball travels?

References

- §8.2 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0–534–41022–7.

Notes

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