Math 1b. Lecture 19
Other Convergence Tests (The Ratio Test)

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1 Goals

- To understand and be able to apply the ration test to determine the convergence or divergence of a series.

2 The Ratio Test

Given the series $\sum_{n=1}^{\infty} a_n$, let

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|.$$

- If $L < 1$, then the series $\sum a_n$ is absolutely convergent.

- If $L > 1$, then the series $\sum a_n$ is diverges.

- If $L = 1$, the *Ratio Test* fails.

3 Worksheet Problems

1. Determine the convergence of $\sum_{n=1}^{\infty} \dfrac{n!}{(2n)!}$.

2. Determine the convergence of $\sum_{n=1}^{\infty} \dfrac{n!}{(n+2)!}$.

3. Determine the convergence of $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt[3]{n^2 - 1}}$.

4. Determine the convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$.

5. Determine the convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^5+4}$.

6. Determine the convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n3^n}$.

7. Determine the convergence of $1 + \dfrac{1}{1 \cdot 3} + \dfrac{1}{1 \cdot 3 \cdot 5} + \dfrac{1}{1 \cdot 3 \cdot 5 \cdot 7} + \cdots$.

References

- §8.4 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0–534–41022–7.

Notes

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