Math 1b. Lecture 20
Strategies for Determining Convergence and Divergence of a Series

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1 Goals

- To understand and be able to apply the different tests for convergence and divergence of a series.

2 Strategies

1. If the series is of the form $\sum 1/n^p$, then it is a p-series. The series converges for $p > 1$ and diverges for $p \leq 1$.
2. If the series has the form $\sum ar^n$, then it is a geometric series and converges for $|r| < 1$ and diverges for $|r| \geq 1$.
3. If the series is similar to a p-series or a geometric series, consider the Comparison Test.
4. If $\lim_{n \to \infty} a_n \neq 0$, then the series diverges.
5. If the series is of the form $\sum (-1)^{n+1} a_n$, consider applying the Alternating Series Test. You can also test for absolute convergence.
6. If the series involves products, factorials, or constants raised to the nth power, consider the Ratio Test.
7. If $a_n = f(n)$ and the integral $\int_1^\infty f(x)\,dx$ is easily evaluated, the Integral Test may be useful assuming the hypothesis of the test are satisfied.

8. Is the series a telescopic series? If so, convergence or divergence can be determined by computing the limit of the partial sums of the series.

3 Worksheet Problems

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln n}}$

4. $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{3^k+4^k}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

6. $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

7. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

8. $\displaystyle\sum_{n=1}^{\infty} \tan(1/n)$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3-2n^2+5}$

10. $\displaystyle\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}}$

References

- §8.2–8.4 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0–534–41022–7.

Notes

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