

# Math 1b. Lecture 25

## Applications of Taylor Polynomials

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### 1 Goals

- To understand how Taylor polynomials can be used to approximate functions.

### Taylor's Remainder Theorem and Taylor's Inequality<sup>1</sup>

- If  $|f^{(n+1)}| \leq M$  between  $a$  and  $x$ , then the remainder of the Taylor series satisfies the inequality

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}.$$

- If  $f$  has continuous derivatives of orders  $1, 2, \dots, n+1$  between  $a$  and  $x$ , then

$$f(x) = T_n(x) + \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1}$$

where  $c$  lies between  $a$  and  $x$ .

Notice that the Mean Value Theorem is a special case of Taylor's Remainder Theorem.

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<sup>1</sup>This material is different than what is presented in Stewart.

## 2 Worksheet Problems

1. Compute the Taylor polynomial of  $f(x) = \sqrt{x}$  for  $n = 2$  and  $a = 4$ . Estimate the accuracy of the approximation  $f(x) \approx T_n(x)$  when  $x$  lies in the interval  $4 \leq x \leq 4.2$ .
2. Compute the Taylor polynomial of  $f(x) = \cos x$  for  $n = 4$  and  $a = \pi/3$ . Estimate the accuracy of the approximation  $f(x) \approx T_n(x)$  when  $x$  lies in the interval  $0 \leq x \leq 2\pi/3$ .
3. Let  $f$  be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for  $f$  about  $x = 2$  is given by

$$T_3(x) = 7 - 9(x - 2)^2 - 3(x - 2)^3.$$

- (a) Find  $f(2)$  and  $f'(2)$ .
- (b) Is there enough information given to determine whether  $f$  has a critical point at  $x = 2$ ? If not, explain why not. If so, determine whether  $f(2)$  is a relative minimum, a relative maximum, or neither. Justify your answer.
- (c) Use  $T_3(x)$  to find an approximation for  $f(0)$ . Is there enough information given to determine whether  $f$  has a critical point at  $x = 0$ ? If not, explain why not. If so, determine whether  $f(0)$  is a relative minimum, a relative maximum, or neither. Justify your answer.
- (d) The fourth derivative of  $f$  satisfies the inequality

$$\left| f^{(4)}(x) \right| \leq 6$$

for all  $x$  in the closed interval  $[0, 2]$ . Find an error bound on the approximation for  $f(0)$  that you found in part (c). Explain why  $f(0)$  must be negative.

## References

- §8.9 in James Stewart. *Single Variable Calculus: Concepts & Context*, third edition. Brooks/Cole, Belmont CA, 2005. ISBN 0-534-41022-7.

## Notes

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