Math 1b. Trigonometric Integrals and Trigonometric Substitution

Spring 2006

Some Useful Identities

$$\cos^2 x + \sin^2 x = 1 \qquad \sec^2 x = \tan^2 x + 1$$
$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Worksheet Exercises

Evaluate each of the following integrals

1. $\int \cos^3 x \sin^4 x \, dx$

2. $\int \sin^2 x \cos^2 x \, dx$

3. $\int \sec^3 x \tan^3 x \, dx.$

4. $\int \sec\theta \, d\theta = \int \frac{\cos\theta}{\cos^2\theta} \, d\theta = \int \frac{\cos\theta}{1-\sin^2\theta} \, d\theta$. *Hint*: Make use of the identity

$$\frac{2}{1-u^2} = \frac{1}{1-u} + \frac{1}{1+u}.$$

5. $\int \sqrt{4-x^2} \, dx$

6. $\int \frac{1}{\sqrt{4+x^2}} \, dx$