

Math 1b. Applying the Definite Integral: Slice and Conquer

Spring 2006

1. A city is in the shape of a rectangle 4 miles wide and 6 miles long. A river runs through the middle of the city, parallel to the 6-mile long sides. People prefer to live nearer the water, so the density of people is given by $\rho(x) = 10000 - 800x$ people per square mile, where x is the distance from the river.
 - (a) Show in a sketch how to slice up the region.
 - (b) What is the area of the i th slice?
 - (c) What is the approximate population of the i th slice?
 - (d) Write a Riemann sum to estimate the total population of the city.
 - (e) Calculate the population of the city using a definite integral.

2. A village is in the shape of a circle of radius 3 kilometers. A well, located in the center of the city, is the only source of water. People prefer to live nearer the well, so the density of people is given by $\rho(x) = 800 - 100x$ people per square mile, where x is the distance from the well.
- Show in a sketch how to slice up the region.
 - What is the area of the i th slice?
 - What is the approximate population of the i th slice?
 - Write a Riemann sum to estimate the total population of the city.
 - Calculate the population of the city using a definite integral.

3. A beam of light is shining onto a screen creating a disk of radius 50 cm. The intensity of the light is brightest at the center and diminishes away from the center. If the intensity of the light at a distance r from the center of the beam is given by

$$f(r) = \frac{150}{20 + r^2}$$

watts per cm^2 , find the total wattage of the beam's image on the screen.

4. Suppose the density of a mass in a gaseous planet is given by the function

$$\rho(r) = \frac{40000}{1 + 0.0001r^3}$$

kilograms per cubic kilometer, where r is the distance in kilometers from the center of the planet. Find the total mass of the planet if its radius is 8000 km.