

# Math 1b. Sequences

Spring 2006

1. Give an example of a sequence that is:

(a) convergent but not monotone.

(b) monotone but not convergent.

(c) bounded but not monotone.

(d) monotone decreasing and unbounded.

(e) monotone increasing and convergent.

(f) unbounded but not monotone.

2. Find  $a_n$  for the sequence  $-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \dots$

3. Compute  $\lim_{k \rightarrow \infty} a_k$  if it exists, where

$$a_k = \left(1 + \frac{2}{k}\right)^k$$

4. Compute  $\lim_{k \rightarrow \infty} a_k$  if it exists, where

$$a_k = \ln k - \ln(3k + 2).$$

5. Show that the sequence defined by

$$a_1 = 2 \quad a_{n+1} = \frac{1}{3 - a_n}$$

satisfies  $0 < a_n \leq 2$  and is decreasing. Deduce that the sequence is convergent and find its limit.