

# Math 1b. Series—Definitions and Properties

Spring 2006

1. Determine the convergence or divergence of each of the following series. If the series converges, try to find the sum.

(a)  $\sum_{n=1}^{\infty} \tan n$

(b)  $\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$

(c)  $\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$

(d)  $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

2. Let  $\{a_n\}$  be an increasing sequence such that  $a_1 > 0$  and  $a_n \leq 100$  for all  $n \geq 1$ .

(a) Does  $\lim_{n \rightarrow \infty} a_n$  converge?

(b) Does  $\sum_{n=1}^{\infty} a_n$  converge?

3. Suppose that the partial sums of  $\sum_{n=1}^{\infty} a_n$  satisfy the inequality

$$\frac{6 \ln n}{\ln(n^2 + 1)} < S_n < 3 + ne^{-n}$$

for all  $n > 100$ .

(a) Does  $\sum_{n=1}^{\infty} a_n$  converge? If so, what is the limit?

(b) Can anything be said about  $\lim_{n \rightarrow \infty} a_n$ ?