Math 1b. Representations of Functions by Power Series

Spring 2006

Function	Series	Interval of Convergence
e^x	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$(-\infty, \infty)$
$\sin x$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$	$(-\infty, \infty)$
$\cos x$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$	$(-\infty, \infty)$
$\frac{1}{1-x}$	$1 + x + x^2 + x^3 + \cdots$	$(-1, 1)$
$\frac{1}{1+x}$	$1 - x + x^2 - x^3 + \cdots$	$(-1, 1)$
$\frac{1}{1+x^2}$	$1 - x^2 + x^4 - x^6 + \cdots$	$(-1, 1)$
$\arctan x$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$	$[-1, 1]$
$\ln(x+1)$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	$(-1, 1)$

1. Find a power series representation for $f(x) = \frac{x^2}{1+x}$.

2. Find a power series representation for $f(x) = \dfrac{1}{(1+x)^2}$.

3. Find a power series representation for $f(x) = \dfrac{1}{2+x}$ and determine the interval of convergence.

4. Find a power series representation for $f(x) = x^2 \cos x$.

5. Find a power series representation for $f(x) = \cos(x^2)$.

6. Show that $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots = \ln 2$.