Math 1b. Taylor and Maclaurin Series I

Spring 2006

- A *Taylor Series* for f centered at $x = a$ is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots .$$

- A Taylor series centered at $x = 0$ is called a *Maclaurin series*,

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

- The nth degree *Taylor Polynomial* of a function f is

$$T_n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

1. Find the Taylor polynomial for $f(x) = \cos x$, where $n = 4$ and $a = 0$

2. Find the Taylor polynomial for $f(x) = \cos x$, where $n = 4$ and $a = \pi/6$

3. Find the Taylor series for $f(x) = \cos x$, where $a = 0$.

4. Find the Taylor series for $f(x) = x^3 - 3x^2 + 2$, where $a = 2$.

5. Find the Taylor series for $f(x) = \ln x$, where $a = 1$.