

Math 1b. Taylor and Maclaurin Series II

Spring 2006

- The n th degree *Taylor Polynomial* of a function f is

$$T_n(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x - a)^n.$$

If f can be represented by a Taylor series, then $f(x) = T_n(x) + R_n(x)$, where R_n is the remainder series.

- If $|f^{(n+1)}| \leq M$ between a and x , then the remainder of the Taylor series satisfies the inequality

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x - a|^{n+1}.$$

1. Let

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots$$

for all real numbers x .

- (a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local minimum, a local maximum, or neither at $x = 0$.

- (b) Show that $y = f(x)$ is a solution to the differential equation

$$xy' + y = \cos x.$$

2. Let f be the function given by

$$f(x) = \sin\left(5x + \frac{\pi}{4}\right),$$

and let $P(x)$ be the third-degree Taylor polynomial for f centered about $x = 0$.

(a) Find $P(x)$.

(b) Find the coefficient of x^{22} in the Taylor series for f about $x = 0$.

(c) Use the error bound to show that

$$\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| < \frac{1}{100}.$$

(d) Let

$$G(x) = \int_0^x f(t) dt.$$

Write the third-degree Taylor polynomial for G about $x = 0$.

3. If $f(x) = \cos x$ and $g(x) = \sin x$, find the first five terms of the power series for $f(x)g(x)$.