Math 1b. Solutions to Differential Equations

Spring 2006

1. Two identical cups of dark liquid are left in a 70°F laboratory cool. At time $t = 0$, the first cup's temperature was 190°F, and was dropping at a rate of 12°F per minute. When did this cup's temperature fall to 130°F? The second cup was at 130°F after 10 minutes. Could this liquid be coffee?

2. Decide whether the given function is a solution to the differential equation.

    (a) $y(t) = t^2/2$; $y' = t$

    (b) $y(t) = t^2/2$; $y' = y$

    (c) $y(t) = \dfrac{1}{2}t^4 + \dfrac{3}{2}t^2 + \dfrac{1}{4}$; $y' - \dfrac{2y}{t} = t^3$

    (d) $y(t) = \dfrac{1}{2}t^4 + \dfrac{3}{2}t^2$; $y' - \dfrac{2y}{t} = t^3$

3. Verify that
$$y(t) = 70 + 120e^{-0.1t}$$
    is a solution to the differential equation
$$\frac{dy}{dt} = -\frac{1}{10}(y - 70),$$
$$y(0) = 190.$$

4. (8 points)   Match equations and slope fields

(a) $\dfrac{dy}{dt} = 1 + y^2$

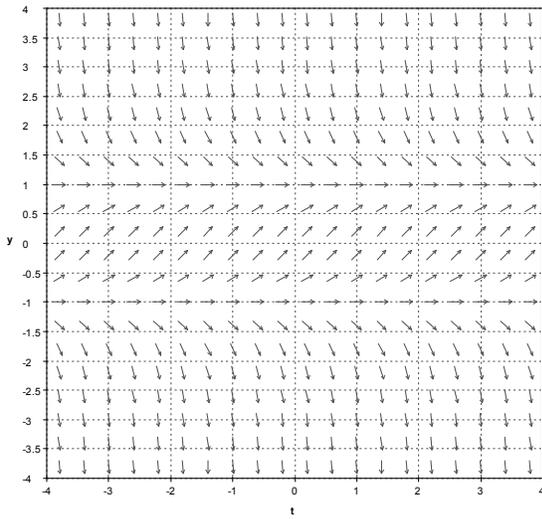(b) $\dfrac{dy}{dt} = y^2 - t^2$

(c) $\dfrac{dy}{dt} = ty$

(d) $\dfrac{dy}{dt} = 1 - y$

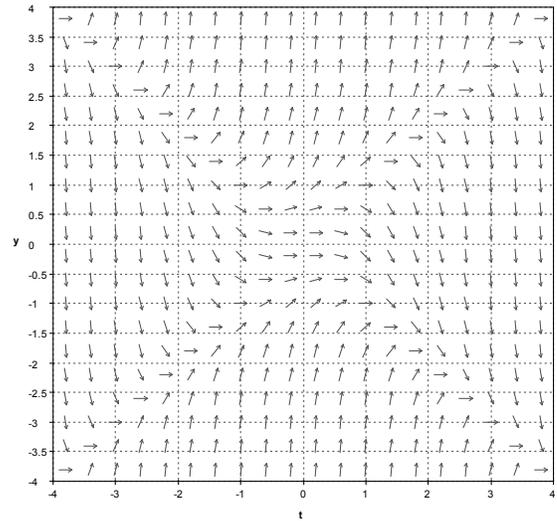(e) $\dfrac{dy}{dt} = y(1 - y) - 2$

(f) $\dfrac{dy}{dt} = (y - t)^2$
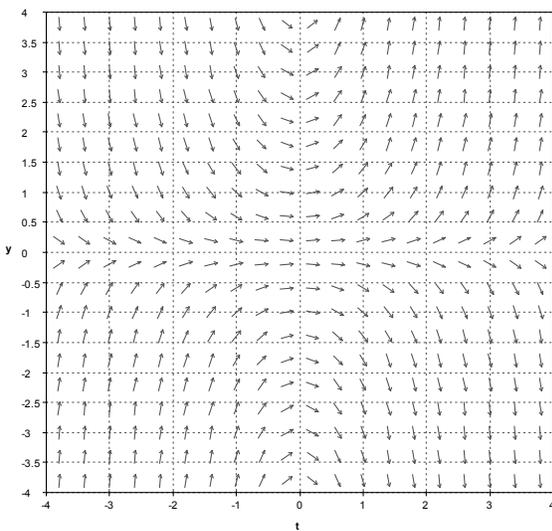
(g) $\dfrac{dy}{dt} = 1 - y^2$
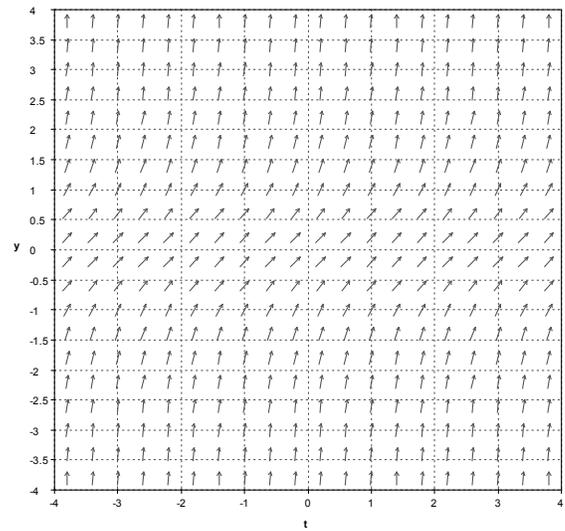
(h) $\dfrac{dy}{dt} = y^2 - 4t^2$

(i)

(ii)

(iii)

(iv)

2