5. Let $u = 1 + 2x$. Then $du = 2\,dx$ and $dx = \frac{1}{2}\,du$, so

$$\int \frac{4}{(1+2x)^3}\,dx = 4\int u^{-3}\left(\tfrac{1}{2}\,du\right) = 2\frac{u^{-2}}{-2} + C = -\frac{1}{u^2} + C = -\frac{1}{(1+2x)^2} + C.$$

6. Let $u = \sin\theta$. Then $du = \cos\theta\,d\theta$, so $\int e^{\sin\theta}\cos\theta\,d\theta = \int e^u\,du = e^u + C = e^{\sin\theta} + C$.

16. Let $u = 5t + 4$. Then $du = 5\,dt$ and $dt = \frac{1}{5}\,du$, so

$$\int \frac{1}{(5t+4)^{2.7}}\,dt = \int u^{-2.7}\left(\tfrac{1}{5}\,du\right) = \frac{1}{5}\cdot\frac{1}{-1.7}u^{-1.7} + C = \frac{-1}{8.5}u^{-1.7} + C = \frac{-2}{17(5t+4)^{1.7}} + C.$$

18. Let $u = 2y^4 - 1$. Then $du = 8y^3\,dy$ and $y^3\,dy = \frac{1}{8}\,du$, so

$$\int y^3\sqrt{2y^4-1}\,dy = \int u^{1/2}\left(\tfrac{1}{8}\,du\right) = \tfrac{1}{8}\cdot\tfrac{2}{3}u^{3/2} + C = \tfrac{1}{12}(2y^4-1)^{3/2} + C.$$

20. Let $u = 2\theta$. Then $du = 2\,d\theta$ and $d\theta = \frac{1}{2}\,du$, so $\int \sec 2\theta\,\tan 2\theta\,d\theta = \int \sec u\tan u\left(\frac{1}{2}\,du\right) = \frac{1}{2}\sec u + C = \frac{1}{2}\sec 2\theta + C$.

30. Let $u = e^x + 1$. Then $du = e^x\,dx$, so $\displaystyle\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x+1) + C$.

47. Let $u = x - 1$, so $u + 1 = x$ and $du = dx$. When $x = 1$, $u = 0$; when $x = 2$, $u = 1$. Thus,

$$\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 (u^{3/2} + u^{1/2})\,du = \left[\tfrac{2}{5}u^{5/2} + \tfrac{2}{3}u^{3/2}\right]_0^1 = \tfrac{2}{5} + \tfrac{2}{3} = \tfrac{16}{15}.$$

53. Let $u = \ln x$, so $du = \dfrac{dx}{x}$. When $x = e$, $u = 1$; when $x = e^4$; $u = 4$. Thus,

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_1^4 u^{-1/2}\,du = 2\left[u^{1/2}\right]_1^4 = 2(2-1) = 2.$$

62. Number of calculators $= x(4) - x(2) = \int_2^4 5000\left[1 - 100(t+10)^{-2}\right]dt$

$$= 5000\left[t + 100(t+10)^{-1}\right]_2^4 = 5000\left[\left(4 + \tfrac{100}{14}\right) - \left(2 + \tfrac{100}{12}\right)\right] \approx 4048$$

64. Let $u = x^2$. Then $du = 2x\,dx$, so $\int_0^3 x f(x^2)\,dx = \int_0^9 f(u)\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}(4) = 2$.

2. Let $u = \theta$, $dv = \cos\theta\,d\theta \quad\Rightarrow\quad du = d\theta$, $v = \sin\theta$. Then by Equation 2,
$\int \theta\cos\theta\,d\theta = \theta\sin\theta - \int \sin\theta\,d\theta = \theta\sin\theta + \cos\theta + C$.

Note: A mnemonic device which is helpful for selecting u when using integration by parts is the LIATE principle of precedence for u:

L̲ogarithmic
I̲nverse trigonometric
A̲lgebraic
T̲rigonometric
E̲xponential

If the integrand has several factors, then we try to choose among them a u which appears as high as possible on the list. For example, in Exercise 3 the integrand is xe^{2x}, which is the product of an algebraic function (x) and an exponential function (e^{2x}). Since A̲lgebraic appears before E̲xponential, we choose $u = x$. Sometimes the integration turns out to be similar regardless of the selection of u and dv, but it is advisable to refer to LIATE when in doubt.