4. Let $u = x$, $dv = e^{-x}\,dx \quad\Rightarrow\quad du = dx$, $v = -e^{-x}$. Then $\int xe^{-x}\,dx = -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + C$.

9. Let $u = \ln(2x+1)$, $dv = dx \quad\Rightarrow\quad du = \dfrac{2}{2x+1}\,dx$, $v = x$. Then

$$\int \ln(2x+1)\,dx = x\ln(2x+1) - \int \frac{2x}{2x+1}\,dx = x\ln(2x+1) - \int \frac{(2x+1)-1}{2x+1}\,dx$$

$$= x\ln(2x+1) - \int\left(1 - \frac{1}{2x+1}\right)dx = x\ln(2x+1) - x + \tfrac{1}{2}\ln(2x+1) + C$$

$$= \tfrac{1}{2}(2x+1)\,\ln(2x+1) - x + C$$

10. Let $u = \ln p$, $dv = p^5\,dp \quad\Rightarrow\quad du = \dfrac{1}{p}\,dp$, $v = \frac{1}{6}p^6$. Then $\int p^5 \ln p\,dp = \frac{1}{6}p^6\ln p - \frac{1}{6}\int p^5\,dp = \frac{1}{6}p^6\ln p - \frac{1}{36}p^6 + C$.

14. First let $u = e^{-\theta}$, $dv = \cos 2\theta\,d\theta \quad\Rightarrow\quad du = -e^{-\theta}\,d\theta$, $v = \frac{1}{2}\sin 2\theta$. Then

$I = \int e^{-\theta}\cos 2\theta\,d\theta = \frac{1}{2}e^{-\theta}\sin 2\theta - \int \frac{1}{2}\sin 2\theta\left(-e^{-\theta}\,d\theta\right) = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\int e^{-\theta}\sin 2\theta\,d\theta$.

Next let $U = e^{-\theta}$, $dV = \sin 2\theta\,d\theta \quad\Rightarrow\quad dU = -e^{-\theta}\,d\theta$, $V = -\frac{1}{2}\cos 2\theta$, so

$\int e^{-\theta}\sin 2\theta\,d\theta = -\frac{1}{2}e^{-\theta}\cos 2\theta - \int\left(-\frac{1}{2}\right)\cos 2\theta\left(-e^{-\theta}\,d\theta\right) = -\frac{1}{2}e^{-\theta}\cos 2\theta - \frac{1}{2}\int e^{-\theta}\cos 2\theta\,d\theta$.

So $I = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\left[\left(-\frac{1}{2}e^{-\theta}\cos 2\theta\right) - \frac{1}{2}I\right] = \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta - \frac{1}{4}I \quad\Rightarrow$

$\frac{5}{4}I = \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1 \quad\Rightarrow\quad I = \frac{4}{5}\left(\frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1\right) = \frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C$.

22. Let $u = r^2$, $dv = \dfrac{r}{\sqrt{4+r^2}}\,dr \quad\Rightarrow\quad du = 2r\,dr$, $v = \sqrt{4+r^2}$. By (6),

$$\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr = \left[r^2\sqrt{4+r^2}\right]_0^1 - 2\int_0^1 r\sqrt{4+r^2}\,dr = \sqrt{5} - \tfrac{2}{3}\left[\left(4+r^2\right)^{3/2}\right]_0^1$$

$$= \sqrt{5} - \tfrac{2}{3}(5)^{3/2} + \tfrac{2}{3}(8) = \sqrt{5}\left(1 - \tfrac{10}{3}\right) + \tfrac{16}{3} = \tfrac{16}{3} - \tfrac{7}{3}\sqrt{5}$$

23. Let $u = (\ln x)^2$, $dv = dx \quad\Rightarrow\quad du = \dfrac{2}{x}\ln x\,dx$, $v = x$. By Formula 6, $I = \int_1^2 (\ln x)^2\,dx = \left[x(\ln x)^2\right]_1^2 - 2\int_1^2 \ln x\,dx$.

To evaluate the last integral, let $U = \ln x$, $dV = dx \quad\Rightarrow\quad dU = \dfrac{1}{x}\,dx$, $V = x$. Thus,

$$I = [x(\ln x)]_1^2 - 2\left([x\ln x]_1^2 - \int_1^2 dx\right) = [x(\ln x) - 2x\ln x + 2x]_1^2$$

$$= \left(2(\ln 2)^2 - 4\ln 2 + 4\right) - (0 - 0 + 2) = 2(\ln 2)^2 - 4\ln 2 + 2$$

25. Let $w = \sqrt{x}$, so that $x = w^2$ and $dx = 2w\,dw$. Thus, $\int \sin\sqrt{x}\,dx = \int 2w\sin w\,dw$. Now use parts with $u = 2w$, $dv = \sin w\,dw$, $du = 2\,dw$, $v = -\cos w$ to get

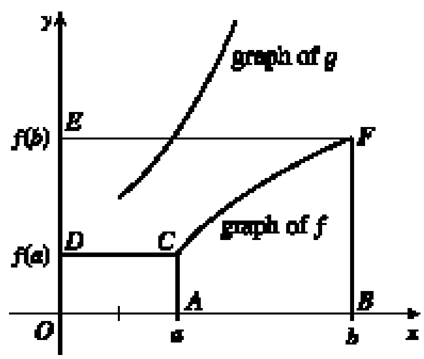
$$\begin{aligned}\int 2w\sin w\,dw &= -2w\cos w + \int 2\cos w\,dw = -2w\cos w + 2\sin w + C \\ &= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C = 2\left(\sin\sqrt{x} - \sqrt{x}\cos\sqrt{x}\right) + C\end{aligned}$$

44. (a) Take $g(x) = x$ and $g'(x) = 1$ in Equation 1.

(b) By part (a), $\int_a^b f(x)\,dx = bf(b) - a\,f(a) - \int_a^b x\,f'(x)\,dx$. Now let $y = f(x)$, so that $x = g(y)$ and $dy = f'(x)\,dx$. Then $\int_a^b x\,f'(x)\,dx = \int_{f(a)}^{f(b)} g(y)\,dy$. The result follows.

(c) Part (b) says that the area of region $ABFC$ is

$$\begin{aligned}&= \quad bf(b) \quad - \quad af(a) \quad - \quad \int_{f(a)}^{f(b)} g(y)\,dy \\ &= (\text{area of rectangle } OBFE) - (\text{area of rectangle } OACD) - (\text{area of region } DCFE)\end{aligned}$$



(d) We have $f(x) = \ln x$, so $f^{-1}(x) = e^x$, and since $g = f^{-1}$, we have $g(y) = e^y$. By part (b),
$\int_1^e \ln x\,dx = e\ln e - 1\ln 1 - \int_{\ln 1}^{\ln e} e^y\,dy = e - \int_0^1 e^y\,dy = e - \left[e^y\right]_0^1 = e - (e-1) = 1.$

1. $\int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \cos^2 x \sin x \, dx = \int (1 - \cos^2 x) \cos^2 x \sin x \, dx \overset{c}{=} \int (1 - u^2) u^2 (-du)$

$= \int (u^2 - 1) u^2 \, du = \int (u^4 - u^2) du = \frac{1}{5} u^5 - \frac{1}{3} u^3 + C = \frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$