2. $\int_0^{\pi/2} \cos^5 x\,dx = \int_0^{\pi/2} (\cos^2 x)^2 \cos x\,dx = \int_0^{\pi/2} (1-\sin^2 x)^2 \cos x\,dx \overset{s}{=} \int_0^1 (1-u^2)^2\,du$

$$= \int_0^1 (1-2u^2+u^4)\,du = \left[u - \tfrac{2}{3}u^3 + \tfrac{1}{5}u^5\right]_0^1 = \left(1 - \tfrac{2}{3} + \tfrac{1}{5}\right) - 0 = \tfrac{8}{15}$$

6. $\int_0^{\pi/2} \sin^2 x\,\cos^2 x\,dx = \int_0^{\pi/2} \frac{1}{4}(4\sin^2 x\,\cos^2 x)\,dx = \int_0^{\pi/2} \frac{1}{4}(2\sin x\,\cos x)^2 dx = \frac{1}{4}\int_0^{\pi/2} \sin^2 2x\,dx$

$$= \tfrac{1}{4}\int_0^{\pi/2} \tfrac{1}{2}(1-\cos 4x)\,dx = \tfrac{1}{8}\int_0^{\pi/2} (1-\cos 4x)\,dx = \tfrac{1}{8}\left[x - \tfrac{1}{4}\sin 4x\right]_0^{\pi/2}$$

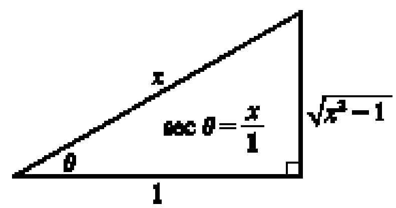
$$= \tfrac{1}{8}\left(\tfrac{\pi}{2}\right) = \tfrac{\pi}{16}$$

7. Let $u = \sec x$. Then $du = \sec x\,\tan x\,dx$, so

$$\int \tan^3 x\,\sec x\,dx = \int (\tan^2 x)(\tan x\,\sec x)\,dx = \int (\sec^2 x - 1)(\sec x\,\tan x\,dx)$$

$$= \int (u^2-1)\,du = \tfrac{1}{3}u^3 - u + C = \tfrac{1}{3}\sec^3 x - \sec x + C$$

10. $x = \sec\theta$, where $0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$. Then $dx = \sec\theta\,\tan\theta\,d\theta$ and $\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = |\tan\theta| = \tan\theta$ (since $\tan\theta \ge 0$ for the specified values of θ). Thus, substitution gives

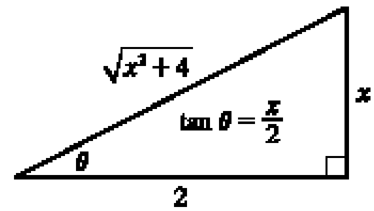


$$\int \frac{\sqrt{x^2-1}}{x^4}\,dx = \int \frac{\tan\theta}{\sec^4\theta}\sec\theta\,\tan\theta\,d\theta = \int \frac{\tan^2\theta}{\sec^3\theta}d\theta = \int \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^3\theta\,d\theta$$

$$= \int \sin^2\theta\,\cos\theta\,d\theta = \int u^2\,du \quad [u = \sin\theta,\ du = \cos\theta\,d\theta]$$

$$= \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3\theta + C = \frac{1}{3}\left(\frac{\sqrt{x^2-1}}{x}\right)^3 + C = \frac{(x^2-1)^{3/2}}{3x^3} + C$$

11. $x = 2\tan\theta$, where $-\pi/2 < \theta < \pi/2$. Then $dx = 2\sec^2\theta\,d\theta$ and



$$\begin{aligned}\sqrt{x^2+4} &= \sqrt{(2\tan\theta)^2+4} = \sqrt{4\tan^2\theta+4} \\ &= \sqrt{4(\tan^2\theta+1)} = 2\sqrt{\sec^2\theta} = 2\,|\sec\theta| \\ &= 2\sec\theta \quad (\text{since } \sec\theta \geq 0 \text{ for } -\pi/2 < \theta < \pi/2).\end{aligned}$$

Thus, substitution gives

$$\begin{aligned}\int \frac{1}{x^2\sqrt{x^2+4}}\,dx &= \int \frac{1}{4\tan^2\theta\,(2\sec\theta)}\left(2\sec^2\theta\,d\theta\right) = \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{4}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\,d\theta \\ &= \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta = \frac{1}{4}\int \frac{1}{u^2}\,du \quad [u = \sin\theta,\ du = \cos\theta\,d\theta] \\ &= \frac{1}{4}\left(-\frac{1}{u}\right) + C = -\frac{1}{4}\,\frac{1}{\sin\theta} + C = -\frac{1}{4}\cdot\frac{\sqrt{x^2+4}}{x} + C = -\frac{\sqrt{x^2+4}}{4x} + C\end{aligned}$$

13. Let $t = \sec\theta$, so $dt = \sec\theta\tan\theta\,d\theta$, $t = \sqrt{2} \Rightarrow \theta = \frac{\pi}{4}$, and $t = 2 \Rightarrow \theta = \frac{\pi}{3}$. Then

$$\begin{aligned}\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\,dt &= \int_{\pi/4}^{\pi/3} \frac{1}{\sec^3\theta\tan\theta}\sec\theta\tan\theta\,d\theta = \int_{\pi/4}^{\pi/3} \frac{1}{\sec^2\theta}\,d\theta = \int_{\pi/4}^{\pi/3} \cos^2\theta\,d\theta \\ &= \int_{\pi/4}^{\pi/3} \tfrac{1}{2}(1+\cos 2\theta)\,d\theta = \tfrac{1}{2}\left[\theta + \tfrac{1}{2}\sin 2\theta\right]_{\pi/4}^{\pi/3} \\ &= \tfrac{1}{2}\left[\left(\tfrac{\pi}{3} + \tfrac{1}{2}\tfrac{\sqrt{3}}{2}\right) - \left(\tfrac{\pi}{4} + \tfrac{1}{2}\cdot 1\right)\right] = \tfrac{1}{2}\left(\tfrac{\pi}{12} + \tfrac{\sqrt{3}}{4} - \tfrac{1}{2}\right) = \tfrac{\pi}{24} + \tfrac{\sqrt{3}}{8} - \tfrac{1}{4}\end{aligned}$$

16. (a) $\dfrac{x-1}{x^3+x^2} = \dfrac{x-1}{x^2(x+1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1}$

(b) $\dfrac{x-1}{x^3+x} = \dfrac{x-1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$