

$$7. f(x) = \sqrt[4]{1+x^2}, \Delta x = \frac{2-0}{8} = \frac{1}{4}$$

$$(a) T_8 = \frac{1}{4 \cdot 2} [f(0) + 2f(\frac{1}{4}) + 2f(\frac{1}{2}) + \cdots + 2f(\frac{3}{2}) + 2f(\frac{7}{4}) + f(2)] \approx 2.413790$$

$$(b) M_8 = \frac{1}{4} [f(\frac{1}{8}) + f(\frac{3}{8}) + \cdots + f(\frac{13}{8}) + f(\frac{15}{8})] \approx 2.411453$$

$$(c) S_8 = \frac{1}{4 \cdot 3} [f(0) + 4f(\frac{1}{4}) + 2f(\frac{1}{2}) + 4f(\frac{3}{4}) + 2f(1) + 4f(\frac{5}{4}) + 2f(\frac{3}{2}) + 4f(\frac{7}{4}) + f(2)] \approx 2.412232$$

$$13. f(x) = \frac{\cos x}{x}, \Delta x = \frac{5-1}{8} = \frac{1}{2}$$

$$(a) T_8 = \frac{1}{2 \cdot 2} [f(1) + 2f(\frac{3}{2}) + 2f(2) + \cdots + 2f(4) + 2f(\frac{9}{2}) + f(5)] \approx -0.495333$$

$$(b) M_8 = \frac{1}{2} [f(\frac{5}{4}) + f(\frac{7}{4}) + f(\frac{9}{4}) + f(\frac{11}{4}) + f(\frac{13}{4}) + f(\frac{15}{4}) + f(\frac{17}{4}) + f(\frac{19}{4})] \approx -0.543321$$

$$(c) S_8 = \frac{1}{2 \cdot 3} [f(1) + 4f(\frac{3}{2}) + 2f(2) + 4f(\frac{5}{2}) + 2f(3) + 4f(\frac{7}{2}) + 2f(4) + 4f(\frac{9}{2}) + f(5)] \approx -0.526123$$

$$18. (a) T_8 = \frac{1}{8 \cdot 2} \{f(0) + 2[f(\frac{1}{8}) + f(\frac{2}{8}) + \cdots + f(\frac{7}{8})] + f(1)\} \approx 0.902333$$

$$M_8 = \frac{1}{8} [f(\frac{1}{16}) + f(\frac{3}{16}) + f(\frac{5}{16}) + \cdots + f(\frac{15}{16})] = 0.905620$$

(b) $f(x) = \cos(x^2)$, $f'(x) = -2x \sin(x^2)$, $f''(x) = -2 \sin(x^2) - 4x^2 \cos(x^2)$. For $0 \leq x \leq 1$, \sin and \cos are positive, so $|f''(x)| = 2 \sin(x^2) + 4x^2 \cos(x^2) \leq 2 \cdot 1 + 4 \cdot 1 \cdot 1 = 6$ since $\sin(x^2) \leq 1$ and $\cos(x^2) \leq 1$ for all x , and $x^2 \leq 1$ for $0 \leq x \leq 1$. So for $n = 8$, we take $K = 6$, $a = 0$, and $b = 1$ in Theorem 3, to get

$$|E_T| \leq 6 \cdot 1^3 / (12 \cdot 8^2) = \frac{1}{128} = 0.0078125 \text{ and } |E_M| \leq \frac{1}{256} = 0.00390625. \text{ [A better estimate is obtained by noting from a graph of } f'' \text{ that } |f''(x)| \leq 4 \text{ for } 0 \leq x \leq 1.]$$

(c) Using $K = 6$ as in part (b), we have $|E_T| \leq 6 \cdot 1^3 / (12n^2) = 1 / (2n^2) \leq 10^{-5} \Rightarrow 2n^2 \geq 10^5 \Rightarrow n \geq \sqrt{\frac{1}{2} \cdot 10^5}$
 or $n \geq 224$. To guarantee that $|E_M| \leq 0.00001$, we need $6 \cdot 1^3 / (24n^2) \leq 10^{-5} \Rightarrow 4n^2 \geq 10^5 \Rightarrow n \geq \sqrt{\frac{1}{4} \cdot 10^5}$
 or $n \geq 159$.

27. By the Net Change Theorem, the increase in velocity is equal to $\int_0^6 a(t) dt$. We use Simpson's Rule with $n = 6$ and $\Delta t = (6 - 0)/6 = 1$ to estimate this integral:

$$\int_0^6 a(t) dt \approx S_6 = \frac{1}{3} [a(0) + 4a(1) + 2a(2) + 4a(3) + 2a(4) + 4a(5) + a(6)] \\ \approx \frac{1}{3} [0 + 4(0.5) + 2(4.1) + 4(9.8) + 2(12.9) + 4(9.5) + 0] = \frac{1}{3} (113.2) = 37.7\bar{3} \text{ ft/s}$$

- 37.** Since the Trapezoidal and Midpoint approximations on the interval $[a, b]$ are the sums of the Trapezoidal and Midpoint approximations on the subintervals $[x_{i-1}, x_i]$, $i = 1, 2, \dots, n$, we can focus our attention on one such interval. The condition $f''(x) < 0$ for $a \leq x \leq b$ means that the graph of f is concave down as in Figure 5. In that figure, T_n is the area of the trapezoid $AQRD$, $\int_a^b f(x) dx$ is the area of the region $AQPRD$, and M_n is the area of the trapezoid $ABCD$, so $T_n < \int_a^b f(x) dx < M_n$. In general, the condition $f'' < 0$ implies that the graph of f on $[a, b]$ lies above the chord joining the points $(a, f(a))$ and $(b, f(b))$. Thus, $\int_a^b f(x) dx > T_n$. Since M_n is the area under a tangent to the graph, and since $f'' < 0$ implies that the tangent lies above the graph, we also have $M_n > \int_a^b f(x) dx$. Thus, $T_n < \int_a^b f(x) dx < M_n$.

1. (a) Since $\int_1^{\infty} x^4 e^{-x^4} dx$ has an infinite interval of integration, it is an improper integral of Type I.
- (b) Since $y = \sec x$ has an infinite discontinuity at $x = \frac{\pi}{2}$, $\int_0^{\pi/2} \sec x dx$ is a Type II improper integral.
- (c) Since $y = \frac{x}{(x-2)(x-3)}$ has an infinite discontinuity at $x = 2$, $\int_0^2 \frac{x}{x^2 - 5x + 6} dx$ is a Type II improper integral.
- (d) Since $\int_{-\infty}^0 \frac{1}{x^2 + 5} dx$ has an infinite interval of integration, it is an improper integral of Type I.