

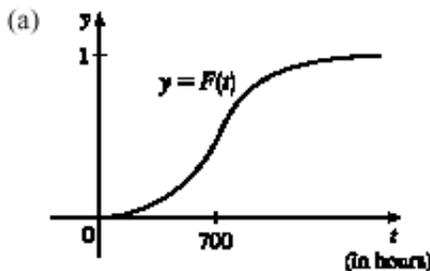
$$8. \int_0^\infty \frac{x}{(x^2 + 2)^2} dx = \lim_{t \rightarrow \infty} \int_0^t \frac{x}{(x^2 + 2)^2} dx = \lim_{t \rightarrow \infty} \frac{1}{2} \left[\frac{-1}{x^2 + 2} \right]_0^t = \frac{1}{2} \lim_{t \rightarrow \infty} \left(\frac{-1}{t^2 + 2} + \frac{1}{2} \right) \\ = \frac{1}{2} \left(0 + \frac{1}{2} \right) = \frac{1}{4}. \quad \text{Convergent}$$

$$14. \int_{-\infty}^\infty x^2 e^{-x^3} dx = \int_{-\infty}^0 x^2 e^{-x^3} dx + \int_0^\infty x^2 e^{-x^3} dx, \text{ and} \\ \int_{-\infty}^0 x^2 e^{-x^3} dx = \lim_{t \rightarrow -\infty} \left[-\frac{1}{3} e^{-x^3} \right]_t^0 = -\frac{1}{3} + \frac{1}{3} \left(\lim_{t \rightarrow -\infty} e^{-t^3} \right) = \infty. \quad \text{Divergent}$$

$$25. \int_{-2}^{14} \frac{dx}{\sqrt[3]{x+2}} = \lim_{t \rightarrow -2^+} \int_t^{14} (x+2)^{-1/4} dx = \lim_{t \rightarrow -2^+} \left[\frac{4}{3} (x+2)^{3/4} \right]_t^{14} = \frac{4}{3} \lim_{t \rightarrow -2^+} [16^{3/4} - (t+2)^{3/4}] \\ = \frac{4}{3} (8 - 0) = \frac{32}{3}. \quad \text{Convergent}$$

$$47. \int_0^\infty \frac{dx}{\sqrt{x}(1+x)} = \int_0^1 \frac{dx}{\sqrt{x}(1+x)} + \int_1^\infty \frac{dx}{\sqrt{x}(1+x)} = \lim_{t \rightarrow 0^+} \int_t^1 \frac{dx}{\sqrt{x}(1+x)} + \lim_{t \rightarrow \infty} \int_1^t \frac{dx}{\sqrt{x}(1+x)}. \text{ Now} \\ \int \frac{dx}{\sqrt{x}(1+x)} = \int \frac{2u du}{u(1+u^2)} \quad [u = \sqrt{x}, x = u^2, dx = 2u du] \\ = 2 \int \frac{du}{1+u^2} = 2 \tan^{-1} u + C = 2 \tan^{-1} \sqrt{x} + C, \\ \text{so } \int_0^\infty \frac{dx}{\sqrt{x}(1+x)} = \lim_{t \rightarrow 0^+} [2 \tan^{-1} \sqrt{x}]_t^1 + \lim_{t \rightarrow \infty} [2 \tan^{-1} \sqrt{x}]_1^t \\ = \lim_{t \rightarrow 0^+} [2(\frac{\pi}{4}) - 2 \tan^{-1} \sqrt{t}] + \lim_{t \rightarrow \infty} [2 \tan^{-1} \sqrt{t} - 2(\frac{\pi}{4})] = \frac{\pi}{2} - 0 + 2(\frac{\pi}{2}) - \frac{\pi}{2} = \pi.$$

53. We would expect a small percentage of bulbs to burn out in the first few hundred hours, most of the bulbs to burn out after close to 700 hours, and a few overachievers to burn on and on.



(b) $r(t) = F'(t)$ is the rate at which the fraction $F(t)$ of burnt-out bulbs increases as t increases. This could be interpreted as a fractional burnout rate. $r(t) = F'(t)$ is the rate at which the fraction $F(t)$ of burnt-out bulbs increases as t increases. This could be interpreted as a fractional burnout rate.

(c) $\int_0^\infty r(t) dt = \lim_{x \rightarrow \infty} F(x) = 1$, since all of the bulbs will eventually burn out.

$$\begin{aligned} \mathbf{1.} \quad A &= \int_{x=0}^{x=4} (y_T - y_B) \, dx = \int_0^4 [(5x - x^2) - x] \, dx = \int_0^4 (4x - x^2) \, dx \\ &= \left[2x^2 - \frac{1}{3}x^3 \right]_0^4 = \left(32 - \frac{64}{3} \right) - (0) = \frac{32}{3} \end{aligned}$$