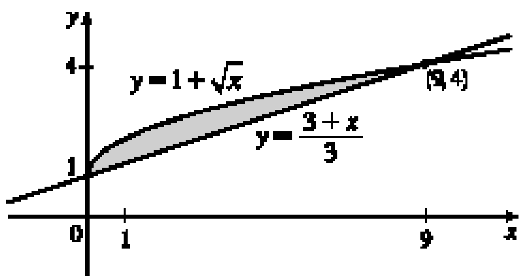
3. $A = \int_{y=-1}^{y=1} (x_R - x_L)\, dy = \int_{-1}^{1} \left[e^y - \left(y^2 - 2\right)\right] dy$

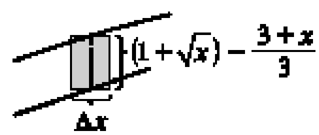
$= \int_{-1}^{1} (e^y - y^2 + 2)\, dy = \left[e^y - \frac{1}{3}y^3 + 2y\right]_{-1}^{1} = \left(e^1 - \frac{1}{3} + 2\right) - \left(e^{-1} + \frac{1}{3} - 2\right) = e - \frac{1}{e} + \frac{10}{3}$

8. $1 + \sqrt{x} = \frac{3+x}{3} = 1 + \frac{x}{3} \quad\Rightarrow\quad \sqrt{x} = \frac{x}{3} \quad\Rightarrow\quad x = \frac{x^2}{9} \quad\Rightarrow\quad 9x - x^2 = 0 \quad\Rightarrow\quad x(9-x) = 0 \quad\Rightarrow\quad x = 0$ or 9, so

$$A = \int_0^9 \left[\left(1 + \sqrt{x}\right) - \left(\frac{3+x}{3}\right)\right] dx = \int_0^9 \left[\left(1 + \sqrt{x}\right) - \left(1 + \frac{x}{3}\right)\right] dx$$

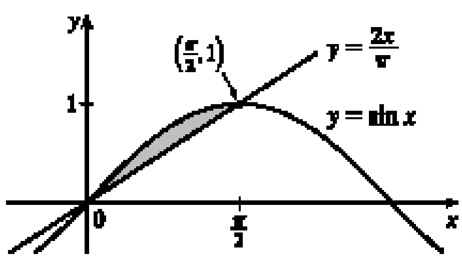
$$= \int_0^9 \left(\sqrt{x} - \tfrac{1}{3}x\right) dx = \left[\tfrac{2}{3}x^{3/2} - \tfrac{1}{6}x^2\right]_0^9 = 18 - \tfrac{27}{2} = \tfrac{9}{2}$$

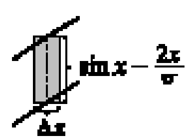




14. By observation, $y = \sin x$ and $y = 2x/\pi$ intersect at $(0, 0)$ and $(\pi/2, 1)$ for $x \geq 0$.

$$A = \int_0^{\pi/2} \left(\sin x - \frac{2x}{\pi}\right) dx = \left[-\cos x - \frac{1}{\pi}x^2\right]_0^{\pi/2} = \left(0 - \frac{\pi}{4}\right) - (-1) = 1 - \frac{\pi}{4}$$





21. As in Example 4, we approximate the distance between the two cars after ten seconds using Simpson's Rule with $\Delta t = 1\text{ s} = \frac{1}{3600}$ h.

$$\text{distance}_{\text{Kelly}} - \text{distance}_{\text{Chris}} = \int_0^{10} v_K\,dt - \int_0^{10} v_C\,dt = \int_0^{10}(v_K - v_C)\,dt \approx S_{10}$$

$$= \tfrac{1}{3\cdot 3600}[(0-0) + 4(22-20) + 2(37-32) + 4(52-46) + 2(61-54) + 4(71-62)$$

$$+ 2(80-69) + 4(86-75) + 2(93-81) + 4(98-86) + (102-90)]$$

$$= \tfrac{1}{10{,}800}(242) = \tfrac{121}{5400}\text{ mi}$$

So after 10 seconds, Kelly's car is about $\dfrac{121}{5400}\text{ mi}\left(5280\dfrac{\text{ft}}{\text{mi}}\right) \approx 118$ ft ahead of Chris's.

22. We know that the area under curve A between $t = 0$ and $t = x$ is $\int_0^x v_A(t)\,dt = s_A(x)$, where $v_A(t)$ is the velocity of car A and s_A is its displacement. Similarly, the area under curve B between $t = 0$ and $t = x$ is $\int_0^x v_B(t)\,dt = s_B(x)$.

(a) After one minute, the area under curve A is greater than the area under curve B. So car A is ahead after one minute.

(b) The area of the shaded region has numerical value $s_A(1) - s_B(1)$, which is the distance by which A is ahead of B after 1 minute.

(c) After two minutes, car B is traveling faster than car A and has gained some ground, but the area under curve A from $t = 0$ to $t = 2$ is still greater than the corresponding area for curve B, so car A is still ahead.

(d) From the graph, it appears that the area between curves A and B for $0 \le t \le 1$ (when car A is going faster), which corresponds to the distance by which car A is ahead, seems to be about 3 squares. Therefore, the cars will be side by side at the time x where the area between the curves for $1 \le t \le x$ (when car B is going faster) is the same as the area for $0 \le t \le 1$. From the graph, it appears that this time is $x \approx 2.2$. So the cars are side by side when $t \approx 2.2$ minutes.

26. The area under $R'(x)$ from $x = 50$ to $x = 100$ represents the change in revenue, and the area under $C'(x)$ from $x = 50$ to $x = 100$ represents the change in cost. The shaded region represents the difference between these two values; that is, the increase in profit as the production level increases from 50 units to 100 units. We use the Midpoint Rule with $n = 5$ and $\Delta x = 10$:

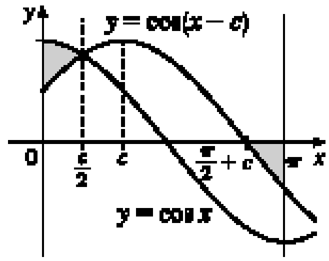
$$M_5 = \Delta x\{[R'(55) - C'(55)] + [R'(65) - C'(65)] + [R'(75) - C'(75)] + [R'(85) - C'(85)] + [R'(95) - C'(95)]\}$$

$$\approx 10(2.40 - 0.85 + 2.20 - 0.90 + 2.00 - 1.00 + 1.80 - 1.10 + 1.70 - 1.20)$$

$$= 10(5.05) = 50.5\text{ thousand dollars}$$

Using M_1 would give us $50(2-1) = 50$ thousand dollars.

42.



It appears from the diagram that the curves $y = \cos x$ and $y = \cos(x - c)$ intersect halfway between 0 and c, namely, when $x = c/2$. We can verify that this is indeed true by noting that $\cos(c/2 - c) = \cos(-c/2) = \cos(c/2)$. The point where $\cos(x - c)$ crosses the x-axis is $x = \frac{\pi}{2} + c$. So we require that $\int_0^{c/2} [\cos x - \cos(x - c)]\,dx = -\int_{\pi/2+c}^{\pi} \cos(x - c)\,dx$ [the negative sign on the RHS is needed since the second area is beneath the x-axis] $\Leftrightarrow$ $[\sin x - \sin(x - c)]_0^{c/2} = -[\sin(x - c)]_{\pi/2+c}^{\pi}$ $\Rightarrow$ $[\sin(c/2) - \sin(-c/2)] - [-\sin(-c)] = -\sin(\pi - c) + \sin\left[\left(\frac{\pi}{2} + c\right) - c\right]$ $\Leftrightarrow$ $2\sin(c/2) - \sin c = -\sin c + 1$. [Here we have used the oddness of the sine function, and the fact that $\sin(\pi - c) = \sin c$]. So $2\sin(c/2) = 1$ $\Leftrightarrow$ $\sin(c/2) = \frac{1}{2}$ $\Leftrightarrow$ $c/2 = \frac{\pi}{6}$ $\Leftrightarrow$ $c = \frac{\pi}{3}$.

2. A cross-section is a disk with radius $1 - x^2$, so its area is $A(x) = \pi(1 - x^2)^2$.

$$V = \int_{-1}^{1} A(x)\,dx = \int_{-1}^{1} \pi(1 - x^2)^2\,dx$$
$$= 2\pi \int_{0}^{1} \left(1 - 2x^2 + x^4\right) dx$$
$$= 2\pi\left[x - \tfrac{2}{3}x^3 + \tfrac{1}{5}x^5\right]_0^1 = 2\pi\left(1 - \tfrac{2}{3} + \tfrac{1}{5}\right)$$
$$= 2\pi\left(\tfrac{8}{15}\right) = \tfrac{16\pi}{15}$$