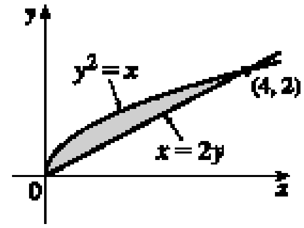
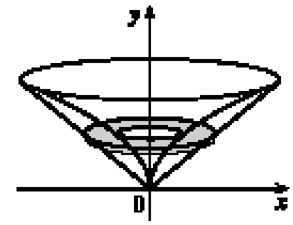
7. A cross-section is a washer with inner radius y^2 and outer radius $2y$, so its area is

$A(y) = \pi(2y)^2 - \pi(y^2)^2 = \pi(4y^2 - y^4)$.

$$V = \int_0^2 A(y)\,dy = \pi \int_0^2 (4y^2 - y^4)\,dy$$

$$= \pi\left[\tfrac{4}{3}y^3 - \tfrac{1}{5}y^5\right]_0^2 = \pi\left(\tfrac{32}{3} - \tfrac{32}{5}\right) = \tfrac{64\pi}{15}$$



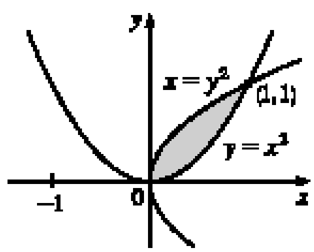


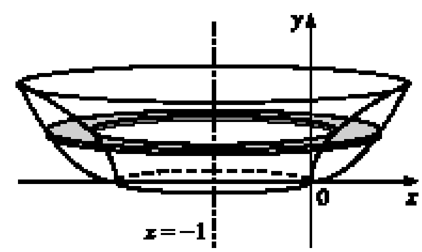
11. $y = x^2 \;\Rightarrow\; x = \sqrt{y}$ for $x \geq 0$. The outer radius is the distance from $x = -1$ to $x = \sqrt{y}$ and the inner radius is the distance from $x = -1$ to $x = y^2$.

$$V = \int_0^1 \pi\left\{[\sqrt{y} - (-1)]^2 - \left[y^2 - (-1)\right]^2\right\} dy = \pi \int_0^1 \left[(\sqrt{y} + 1)^2 - \left(y^2 + 1\right)^2\right] dy$$

$$= \pi \int_0^1 \left(y + 2\sqrt{y} + 1 - y^4 - 2y^2 - 1\right) dy = \pi \int_0^1 \left(y + 2\sqrt{y} - y^4 - 2y^2\right) dy$$

$$= \pi\left[\tfrac{1}{2}y^2 + \tfrac{4}{3}y^{3/2} - \tfrac{1}{5}y^5 - \tfrac{2}{3}y^3\right]_0^1 = \pi\left(\tfrac{1}{2} + \tfrac{4}{3} - \tfrac{1}{5} - \tfrac{2}{3}\right) = \tfrac{29}{30}\pi$$





22. (a) $\pi \int_2^5 y\,dy = \pi \int_2^5 \left(\sqrt{y}\right)^2 dy$ describes the volume of the solid obtained by rotating the region $\mathfrak{R} = \left\{(x, y) \mid 2 \leq y \leq 5, 0 \leq x \leq \sqrt{y}\right\}$ of the xy-plane about the y-axis.

(b) $\pi \int_0^{\pi/2} \left[(1 + \cos x)^2 - 1^2\right] dx$ describes the volume of the solid obtained by rotating the region $\mathfrak{R} = \left\{(x, y) \mid 0 \leq x \leq \frac{\pi}{2}, 1 \leq y \leq 1 + \cos x\right\}$ of the xy-plane about the x-axis.

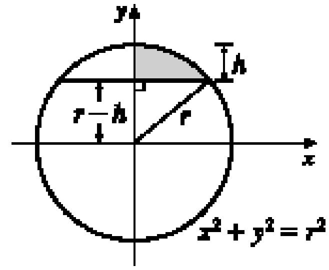
Or: The solid could be obtained by rotating the region $\mathfrak{R}' = \left\{(x, y) \mid 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \cos x\right\}$ about the line $y = -1$.

23. There are 10 subintervals over the 15-cm length, so we'll use $n = 10/2 = 5$ for the Midpoint Rule.

$$V = \int_0^{15} A(x)\,dx \approx M_5 = \tfrac{15-0}{5}[A(1.5) + A(4.5) + A(7.5) + A(10.5) + A(13.5)]$$

$$= 3(18 + 79 + 106 + 128 + 39) = 3 \cdot 370 = 1110 \text{ cm}^3$$

27. $x^2 + y^2 = r^2 \quad \Leftrightarrow \quad x^2 = r^2 - y^2$



$$V = \pi \int_{r-h}^{r} \left(r^2 - y^2\right) dy = \pi \left[r^2 y - \frac{y^3}{3}\right]_{r-h}^{r}$$

$$= \pi \left\{ \left[r^3 - \frac{r^3}{3}\right] - \left[r^2(r-h) - \frac{(r-h)^3}{3}\right] \right\}$$

$$= \pi \left\{ \tfrac{2}{3} r^3 - \tfrac{1}{3}(r-h)\left[3r^2 - (r-h)^2\right] \right\}$$

$$= \tfrac{1}{3}\pi \left\{ 2r^3 - (r-h)\left[3r^2 - \left(r^2 - 2rh + h^2\right)\right] \right\}$$

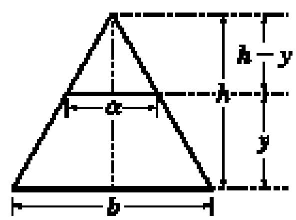
$$= \tfrac{1}{3}\pi \left\{ 2r^3 - (r-h)\left[2r^2 + 2rh - h^2\right] \right\}$$

$$= \tfrac{1}{3}\pi \left(2r^3 - 2r^3 - 2r^2 h + rh^2 + 2r^2 h + 2rh^2 - h^3\right)$$

$$= \tfrac{1}{3}\pi \left(3rh^2 - h^3\right) = \tfrac{1}{3}\pi h^2 (3r - h), \text{ or, equivalently, } \pi h^2 \left(r - \frac{h}{3}\right)$$

29. For a cross-section at height y, we see from similar triangles that $\dfrac{\alpha/2}{b/2} = \dfrac{h-y}{h}$, so $\alpha = b\left(1 - \dfrac{y}{h}\right)$.

Similarly, for cross-sections having $2b$ as their base and β replacing α, $\beta = 2b\left(1 - \dfrac{y}{h}\right)$. So



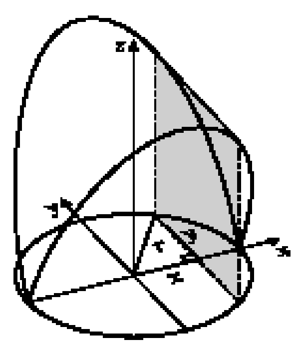
$$V = \int_0^h A(y)\, dy = \int_0^h \left[b\left(1 - \frac{y}{h}\right)\right]\left[2b\left(1 - \frac{y}{h}\right)\right] dy$$

$$= \int_0^h 2b^2 \left(1 - \frac{y}{h}\right)^2 dy = 2b^2 \int_0^h \left(1 - \frac{2y}{h} + \frac{y^2}{h^2}\right) dy$$

$$= 2b^2 \left[y - \frac{y^2}{h} + \frac{y^3}{3h^2}\right]_0^h = 2b^2 \left[h - h + \tfrac{1}{3}h\right]$$

$$= \tfrac{2}{3} b^2 h \quad \left[= \tfrac{1}{3} Bh \text{ where } B \text{ is the area of the base, as with any pyramid.}\right]$$

32. A cross-section is shaded in the diagram.



$A(x) = (2y)^2 = \left(2\sqrt{r^2 - x^2}\right)^2$, so

$$V = \int_{-r}^{r} A(x)\, dx = 2 \int_0^r 4\left(r^2 - x^2\right) dx$$

$$= 8\left[r^2 x - \tfrac{1}{3} x^3\right]_0^r$$

$$= 8\left(\tfrac{2}{3} r^3\right) = \tfrac{16}{3} r^3$$