3. $y = \cos x \quad \Rightarrow \quad dy/dx = -\sin x \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + \sin^2 x$. So $L = \int_0^{2\pi} \sqrt{1 + \sin^2 x}\, dx$.

11. $y = xe^{-x} \quad \Rightarrow \quad dy/dx = e^{-x} - xe^{-x} = e^{-x}(1 - x) \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + e^{-2x}(1 - x)^2$. Let $f(x) = \sqrt{1 + (dy/dx)^2} = \sqrt{1 + e^{-2x}(1 - x)^2}$. Then $L = \int_0^5 f(x)\, dx$. Since $n = 10$, $\Delta x = \frac{5 - 0}{10} = \frac{1}{2}$. Now

$$L \approx S_{10} = \frac{1/2}{3}\left[f(0) + 4f\left(\tfrac{1}{2}\right) + 2f(1) + 4f\left(\tfrac{3}{2}\right) + 2f(2) + 4f\left(\tfrac{5}{2}\right) + 2f(3) + 4f\left(\tfrac{7}{2}\right) + 2f(4) + 4f\left(\tfrac{9}{2}\right) + f(5)\right] \approx 5.115840$$

The value of the integral produced by a calculator is 5.113568 (to six decimal places).

23. The sine wave has amplitude 1 and period 14, since it goes through two periods in a distance of 28 in., so its equation is $y = 1\sin\left(\frac{2\pi}{14}x\right) = \sin\left(\frac{\pi}{7}x\right)$. The width w of the flat metal sheet needed to make the panel is the arc length of the sine curve from $x = 0$ to $x = 28$. We set up the integral to evaluate w using the arc length formula with $\frac{dy}{dx} = \frac{\pi}{7}\cos\left(\frac{\pi}{7}x\right)$: $L = \int_0^{28} \sqrt{1 + \left[\frac{\pi}{7}\cos\left(\frac{\pi}{7}x\right)\right]^2}\, dx = 2\int_0^{14} \sqrt{1 + \left[\frac{\pi}{7}\cos\left(\frac{\pi}{7}x\right)\right]^2}\, dx$. This integral would be very difficult to evaluate exactly, so we use a CAS, and find that $L \approx 29.36$ inches.

6. (a) $f_{\text{ave}} = \frac{1}{3-1}\int_1^3 \ln x\,dx = \frac{1}{2}[x\ln x - x]_1^3$ [by parts]

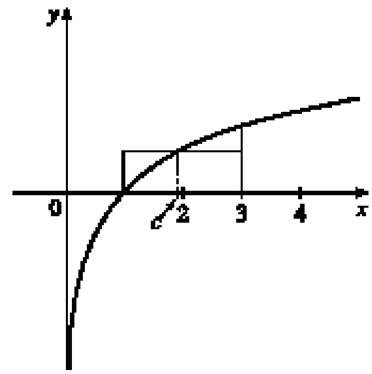
$= \frac{1}{2}[(3\ln 3 - 3) - (\ln 1 - 1)]$

$= \frac{1}{2}(3\ln 3 - 2) = \frac{3}{2}\ln 3 - 1$

(b) $f_{\text{ave}} = f(c) \quad \Leftrightarrow \quad \frac{3}{2}\ln 3 - 1 = \ln c \quad \Leftrightarrow$

$c = e^{(3/2)\ln 3 - 1}$ or $c = 3\sqrt{3}/e \approx 1.91$

(c)



11. $f_{\text{ave}} = \dfrac{1}{50-20}\displaystyle\int_{20}^{50} f(x)\,dx \approx \frac{1}{30}M_3 = \dfrac{1}{30}\cdot\dfrac{50-20}{3}[f(25)+f(35)+f(45)] = \frac{1}{3}(38+29+48) = \frac{115}{3} = 38\frac{1}{3}$

13. Let $t = 0$ and $t = 12$ correspond to 9 A.M. and 9 P.M., respectively.

$$T_{\text{ave}} = \frac{1}{12-0}\int_0^{12}\left[50 + 14\sin\tfrac{1}{12}\pi t\right]dt = \frac{1}{12}\left[50t - 14\cdot\tfrac{12}{\pi}\cos\tfrac{1}{12}\pi t\right]_0^{12}$$

$$= \frac{1}{12}\left[50\cdot 12 + 14\cdot\tfrac{12}{\pi} + 14\cdot\tfrac{12}{\pi}\right] = \left(50 + \tfrac{28}{\pi}\right)\,°\text{F} \approx 59\,°\text{F}$$

15. (a) We want to calculate the square root of the average value of $[E(t)]^2 = [155\sin(120\pi t)]^2 = 155^2\sin^2(120\pi t)$. First, we calculate the average value itself, by integrating $[E(t)]^2$ over one cycle (between $t = 0$ and $t = \frac{1}{60}$, since there are 60 cycles per second) and dividing by $\left(\frac{1}{60} - 0\right)$:

$$[E(t)]^2_{\text{ave}} = \frac{1}{1/60}\int_0^{1/60}\left[155^2\sin^2(120\pi t)\right]dt = 60\cdot 155^2\int_0^{1/60}\tfrac{1}{2}[1 - \cos(240\pi t)]\,dt$$

$$= 60\cdot 155^2\left(\tfrac{1}{2}\right)\left[t - \tfrac{1}{240\pi}\sin(240\pi t)\right]_0^{1/60} = 60\cdot 155^2\left(\tfrac{1}{2}\right)\left[\left(\tfrac{1}{60} - 0\right) - (0 - 0)\right] = \frac{155^2}{2}$$

The RMS value is just the square root of this quantity, which is $\frac{155}{\sqrt{2}} \approx 110$ V.

2. $W = \int_1^2 \cos\left(\frac{1}{3}\pi x\right)\,dx = \frac{3}{\pi}\left[\sin\left(\frac{1}{3}\pi x\right)\right]_1^2 = \frac{3}{\pi}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) = 0\ \text{N}\cdot\text{m} = 0\ \text{J}.$

Interpretation: From $x = 1$ to $x = \frac{3}{2}$, the force does work equal to $\int_1^{3/2} \cos\left(\frac{1}{3}\pi x\right)dx = \frac{3}{\pi}\left(1 - \frac{\sqrt{3}}{2}\right)$ J in accelerating the particle and increasing its kinetic energy. From $x = \frac{3}{2}$ to $x = 2$, the force opposes the motion of the particle, decreasing its kinetic energy. This is negative work, equal in magnitude but opposite in sign to the work done from $x = 1$ to $x = \frac{3}{2}$.