3. The force function is given by $F(x)$ (in newtons) and the work (in joules) is the area under the curve, given by $\int_0^8 F(x)\,dx = \int_0^4 F(x)\,dx + \int_4^8 F(x)\,dx = \frac{1}{2}(4)(30) + (4)(30) = 180$ J.

6. $25 = f(x) = kx = k(0.1)$ [10 cm = 0.1 m], so $k = 250$ N/m and $f(x) = 250x$. Now 5 cm = 0.05 m, so $W = \int_0^{0.05} 250x\,dx = \left[125x^2\right]_0^{0.05} = 125(0.0025) = 0.3125 \approx 0.31$ J.

In Exercises 9–16, n is the number of subintervals of length Δx, and x_i^* is a sample point in the ith subinterval $[x_{i-1}, x_i]$.

10. *Assumptions*:

1. After lifting, the chain is L-shaped, with 4 m of the chain lying along the ground.
2. The chain slides effortlessly and without friction along the ground while its end is lifted.
3. The weight density of the chain is constant throughout its length and therefore equals $(8\text{ kg/m})(9.8\text{ m/s}^2) = 78.4\text{ N/m}$.

The part of the chain x m from the lifted end is raised $6 - x$ m if $0 \le x \le 6$ m, and it is lifted 0 m if $x > 6$ m.

Thus, the work needed is

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} (6 - x_i^*) \cdot 78.4\,\Delta x = \int_0^6 (6 - x) 78.4\,dx = 78.4\left[6x - \tfrac{1}{2}x^2\right]_0^6 = (78.4)(18) = 1411.2\text{ J}$$

In Exercises 9–16, n is the number of subintervals of length Δx, and x_i^* is a sample point in the ith subinterval $[x_{i-1}, x_i]$.

12. The work needed to lift the bucket itself is 4 lb · 80 ft = 320 ft-lb. At time t (in seconds) the bucket is $x_i^* = 2t$ ft above its original 80 ft depth, but it now holds only $(40 - 0.2t)$ lb of water. In terms of distance, the bucket holds $\left[40 - 0.2\left(\frac{1}{2}x_i^*\right)\right]$ lb of water when it is x_i^* ft above its original 80 ft depth. Moving this amount of water a distance Δx requires $\left(40 - \frac{1}{10}x_i^*\right)\Delta x$ ft-lb of work. Thus, the work needed to lift the water is

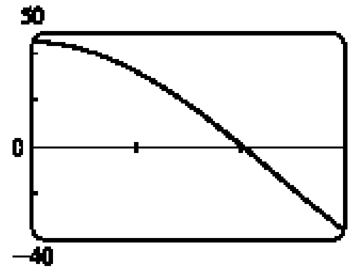
$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \left(40 - \tfrac{1}{10}x_i^*\right)\Delta x = \int_0^{80} \left(40 - \tfrac{1}{10}x\right)dx = \left[40x - \tfrac{1}{20}x^2\right]_0^{80} = (3200 - 320)\text{ ft-lb}$$

Adding the work of lifting the bucket gives a total of 3200 ft-lb of work.

17. (a) A rectangular "slice" of water Δx m thick and lying x ft above the bottom has width x ft and volume $8x\,\Delta x$ m^3. It weighs about $(9.8\times1000)(8x\,\Delta x)$ N, and must be lifted $(5-x)$ m by the pump, so the work needed is about $(9.8\times10^3)(5-x)(8x\,\Delta x)$J. The total work required is

$$W\approx\int_0^3(9.8\times10^3)(5-x)8x\,dx=(9.8\times10^3)\int_0^3(40x-8x^2)\,dx=(9.8\times10^3)\left[20x^2-\tfrac{8}{3}x^3\right]_0^3$$
$$=(9.8\times10^3)(180-72)=(9.8\times10^3)(108)=1058.4\times10^3\approx1.06\times10^6\text{ J}$$

(b) If only 4.7×10^5 J of work is done, then only the water above a certain level (call it h) will be pumped out. So we use the same formula as in part (a), except that the work is fixed, and we are trying to find the lower limit of integration:



$4.7\times10^5\approx\int_h^3(9.8\times10^3)(5-x)8x\,dx=\left(9.8\times10^3\right)\left[20x^2-\frac{8}{3}x^3\right]_h^3 \quad\Leftrightarrow$

$\frac{4.7}{9.8}\times10^2\approx48=\left(20\cdot3^2-\frac{8}{3}\cdot3^3\right)-\left(20h^2-\frac{8}{3}h^3\right)\quad\Leftrightarrow$

$2h^3-15h^2+45=0$. To find the solution of this equation, we plot $2h^3-15h^2+45$ between $h=0$ and $h=3$. We see that the equation is satisfied for $h\approx2.0$. So the depth of water remaining in the tank is about 2.0 m.

21. (a) $W=\displaystyle\int_a^b F(r)\,dr=\int_a^b G\frac{m_1m_2}{r^2}\,dr=Gm_1m_2\left[\frac{-1}{r}\right]_a^b=Gm_1m_2\left(\frac{1}{a}-\frac{1}{b}\right)$

(b) By part (a), $W=GMm\left(\dfrac{1}{R}-\dfrac{1}{R+1{,}000{,}000}\right)$ where $M=$ mass of Earth in kg, $R=$ radius of Earth in m, and $m=$ mass of satellite in kg. (Note that 1000 km $=$ 1,000,000 m.) Thus,

$$W=\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1000)\times\left(\frac{1}{6.37\times10^6}-\frac{1}{7.37\times10^6}\right)\approx8.50\times10^9\text{ J}$$

1. (a) A sequence is an ordered list of numbers. It can also be defined as a function whose domain is the set of positive integers.

(b) The terms a_n approach 8 as n becomes large. In fact, we can make a_n as close to 8 as we like by taking n sufficiently large.

(c) The terms a_n become large as n becomes large. In fact, we can make a_n as large as we like by taking n sufficiently large.