33. $3.\overline{417} = 3 + \frac{417}{10^3} + \frac{417}{10^6} + \cdots$. Now $\frac{417}{10^3} + \frac{417}{10^6} + \cdots$ is a geometric series with $a = \frac{417}{10^3}$ and $r = \frac{1}{10^3}$. It converges to

$\frac{a}{1-r} = \frac{417/10^3}{1 - 1/10^3} = \frac{417/10^3}{999/10^3} = \frac{417}{999}$. Thus, $3.\overline{417} = 3 + \frac{417}{999} = \frac{3414}{999} = \frac{1138}{333}$.

36. $\sum_{n=0}^{\infty} 2^n (x+1)^n = \sum_{n=0}^{\infty} [2(x+1)]^n = \sum_{n=1}^{\infty} [2(x+1)]^{n-1}$ is a geometric series with $r = 2(x+1)$, so the series

converges $\Leftrightarrow \quad |r| < 1 \quad \Leftrightarrow \quad |2(x+1)| < 1 \quad \Leftrightarrow \quad |x+1| < \frac{1}{2} \quad \Leftrightarrow \quad -\frac{1}{2} < x + 1 < \frac{1}{2} \quad \Leftrightarrow \quad -\frac{3}{2} < x < -\frac{1}{2}$.

In that case, the sum of the series is $\frac{a}{1-r} = \frac{1}{1 - 2(x+1)} = \frac{1}{-1 - 2x}$ or $\frac{-1}{2x+1}$.

44. (a) Initially, the ball falls a distance H, then rebounds a distance rH, falls rH, rebounds r^2H, falls r^2H, etc. The total distance it travels is

$$H + 2rH + 2r^2H + 2r^3H + \cdots = H\left(1 + 2r + 2r^2 + 2r^3 + \cdots\right) = H\left[1 + 2r\left(1 + r + r^2 + \cdots\right)\right]$$
$$= H\left[1 + 2r\left(\frac{1}{1-r}\right)\right] = H\left(\frac{1+r}{1-r}\right) \text{ meters}$$

(b) From Example 3 in Section 2.1, we know that a ball falls $\frac{1}{2}gt^2$ meters in t seconds, where g is the gravitational acceleration. Thus, a ball falls h meters in $t = \sqrt{2h/g}$ seconds. The total travel time in seconds is

$$\sqrt{\frac{2H}{g}} + 2\sqrt{\frac{2H}{g}r} + 2\sqrt{\frac{2H}{g}r^2} + 2\sqrt{\frac{2H}{g}r^3} + \cdots = \sqrt{\frac{2H}{g}}\left[1 + 2\sqrt{r} + 2\sqrt{r}^{\,2} + 2\sqrt{r}^{\,3} + \cdots\right]$$
$$= \sqrt{\frac{2H}{g}}\left(1 + 2\sqrt{r}\left[1 + \sqrt{r} + \sqrt{r}^2 + \cdots\right]\right) = \sqrt{\frac{2H}{g}}\left[1 + 2\sqrt{r}\left(\frac{1}{1-\sqrt{r}}\right)\right] = \sqrt{\frac{2H}{g}}\,\frac{1+\sqrt{r}}{1-\sqrt{r}}$$

(c) It will help to make a chart of the time for each descent and each rebound of the ball, together with the velocity just before and just after each bounce. Recall that the time in seconds needed to fall h meters is $\sqrt{2h/g}$. The ball hits the ground with velocity $-g\sqrt{2h/g} = -\sqrt{2hg}$ (taking the upward direction to be positive) and rebounds with velocity $kg\sqrt{2h/g} = k\sqrt{2hg}$, taking time $k\sqrt{2h/g}$ to reach the top of its bounce, where its velocity is 0. At that point, its height is k^2h. All these results follow from the formulas for vertical motion with gravitational acceleration $-g$: $\frac{d^2y}{dt^2} = -g \quad \Rightarrow \quad v = \frac{dy}{dt} = v_0 - gt \quad \Rightarrow \quad y = y_0 + v_0 t - \frac{1}{2}gt^2$.

number of descent	time of descent	speed before bounce	speed after bounce	time of ascent	peak height
1	$\sqrt{2H/g}$	$\sqrt{2Hg}$	$k\sqrt{2Hg}$	$k\sqrt{2H/g}$	k^2H
2	$\sqrt{2k^2H/g}$	$\sqrt{2k^2Hg}$	$k\sqrt{2k^2Hg}$	$k\sqrt{2k^2H/g}$	k^4H
3	$\sqrt{2k^4H/g}$	$\sqrt{2k^4Hg}$	$k\sqrt{2k^4Hg}$	$k\sqrt{2k^4H/g}$	k^6H
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

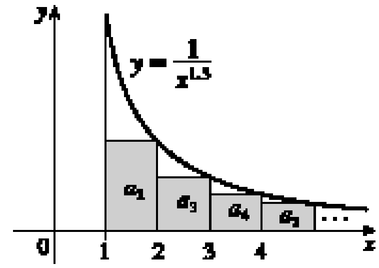
The total travel time in seconds is

$$\sqrt{\frac{2H}{g}} + k\sqrt{\frac{2H}{g}} + k\sqrt{\frac{2H}{g}} + k^2\sqrt{\frac{2H}{g}} + k^2\sqrt{\frac{2H}{g}} + \cdots = \sqrt{\frac{2H}{g}}\left(1 + 2k + 2k^2 + 2k^3 + \cdots\right)$$

$$= \sqrt{\frac{2H}{g}}\left[1 + 2k(1 + k + k^2 + \cdots)\right] = \sqrt{\frac{2H}{g}}\left[1 + 2k\left(\frac{1}{1-k}\right)\right] = \sqrt{\frac{2H}{g}}\,\frac{1+k}{1-k}$$

Another method: We could use part (b). At the top of the bounce, the height is $k^2h = rh$, so $\sqrt{r} = k$ and the result follows from part (b).

45. $\sum_{n=2}^{\infty}(1+c)^{-n}$ is a geometric series with $a = (1+c)^{-2}$ and $r = (1+c)^{-1}$, so the series converges when $\left|(1+c)^{-1}\right| < 1$ $\Leftrightarrow \quad |1+c| > 1 \quad \Leftrightarrow \quad 1+c > 1$ or $1+c < -1 \quad \Leftrightarrow \quad c > 0$ or $c < -2$. We calculate the sum of the series and set it equal to 2: $\frac{(1+c)^{-2}}{1-(1+c)^{-1}} = 2 \quad \Leftrightarrow \quad \left(\frac{1}{1+c}\right)^2 = 2 - 2\left(\frac{1}{1+c}\right) \quad \Leftrightarrow \quad 1 = 2(1+c)^2 - 2(1+c) \quad \Leftrightarrow \quad 2c^2 + 2c - 1 = 0$ $\Leftrightarrow \quad c = \frac{-2 \pm \sqrt{12}}{4} = \frac{\pm\sqrt{3}-1}{2}$. However, the negative root is inadmissible because $-2 < \frac{-\sqrt{3}-1}{2} < 0$. So $c = \frac{\sqrt{3}-1}{2}$.



1. The picture shows that $a_2 = \frac{1}{2^{1.3}} < \int_1^2 \frac{1}{x^{1.3}}\,dx$,

$a_3 = \frac{1}{3^{1.3}} < \int_2^3 \frac{1}{x^{1.3}}\,dx$, and so on, so $\sum_{n=2}^{\infty} \frac{1}{n^{1.3}} < \int_1^{\infty} \frac{1}{x^{1.3}}\,dx$. The integral converges by (5.10.2) with $p = 1.3 > 1$, so the series converges.