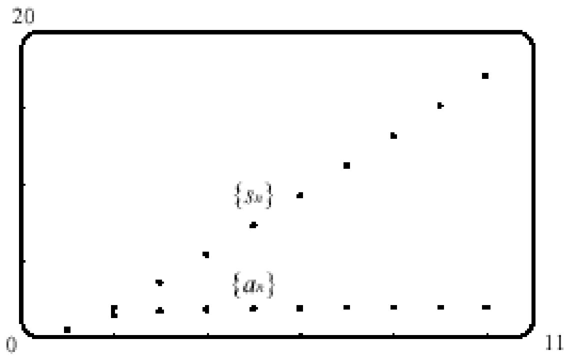
2. $\sum_{n=1}^{\infty} a_n = 5$ means that by adding sufficiently many terms of the series we can get as close as we like to the number 5.

In other words, it means that $\lim_{n\to\infty} s_n = 5$, where s_n is the nth partial sum, that is, $\sum_{i=1}^{n} a_i$.

4.

n	s_n
1	0.50000
2	1.90000
3	3.60000
4	5.42353
5	7.30814
6	9.22706
7	11.16706
8	13.12091
9	15.08432
10	17.05462



The series $\sum_{n=1}^{\infty} \frac{2n^2-1}{n^2+1}$ diverges, since its terms do not approach 0.

9. (a) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2n}{3n+1} = \frac{2}{3}$, so the *sequence* $\{a_n\}$ is convergent by (8.1.1).

(b) Since $\lim_{n\to\infty} a_n = \frac{2}{3} \neq 0$, the *series* $\sum_{n=1}^{\infty} a_n$ is divergent by the Test for Divergence (7).

18. $\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$ diverges since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{2n-3} = \frac{1}{2} \neq 0$. [Use (7), the Test for Divergence.]

27. Using partial fractions, the partial sums of the series $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ are

$$s_n = \sum_{i=2}^{n} \frac{2}{(i-1)(i+1)} = \sum_{i=2}^{n} \left(\frac{1}{i-1} - \frac{1}{i+1}\right)$$
$$= \left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{n-3} - \frac{1}{n-1}\right) + \left(\frac{1}{n-2} - \frac{1}{n}\right)$$

This sum is a telescoping series and $s_n = 1 + \frac{1}{2} - \frac{1}{n-1} - \frac{1}{n}$.

Thus, $\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \left(1 + \frac{1}{2} - \frac{1}{n-1} - \frac{1}{n}\right) = \frac{3}{2}$.