4. $-\frac{1}{3}+\frac{2}{4}-\frac{3}{5}+\frac{4}{6}-\frac{5}{7}+\cdots=\sum_{n=1}^{\infty}(-1)^n\frac{n}{n+2}$. Here $a_n=(-1)^n\frac{n}{n+2}$. Since $\lim_{n\to\infty}a_n\neq 0$ (in fact the limit does not exist), the series diverges by the Test for Divergence.

6. $\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{1+2\sqrt{n}}=\sum_{n=1}^{\infty}(-1)^n b_n$. Now $\lim_{n\to\infty}b_n=\lim_{n\to\infty}\frac{1}{2+1/\sqrt{n}}=\frac{1}{2}\neq 0$. Since $\lim_{n\to\infty}a_n\neq 0$ (in fact the limit does not exist), the series diverges by the Test for Divergence.

8. $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{\ln n}{n}\right)=0+\sum_{n=2}^{\infty}(-1)^{n-1}\left(\frac{\ln n}{n}\right)$. $b_n=\frac{\ln n}{n}>0$ for $n\geq 2$, and if $f(x)=\frac{\ln x}{x}$, then

$f'(x)=\frac{1-\ln x}{x^2}<0$ for $x>e$, so $\{b_n\}$ is eventually decreasing. Also,

$\lim_{n\to\infty}b_n=\lim_{n\to\infty}\frac{\ln n}{n}=\lim_{x\to\infty}\frac{\ln x}{x}\overset{\text{H}}{=}\lim_{x\to\infty}\frac{1/x}{1}=0$, so the series converges by the Alternating Series Test.

9. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{49}-\frac{1}{50}+\frac{1}{51}-\frac{1}{52}+\cdots$. The 50th partial sum of this series is an underestimate, since $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=s_{50}+\left(\frac{1}{51}-\frac{1}{52}\right)+\left(\frac{1}{53}-\frac{1}{54}\right)+\cdots$, and the terms in parentheses are all positive. The result can be seen geometrically in Figure 1.

14. Using the Ratio Test with the series $\sum_{n=1}^{\infty}(-1)^{n-1}ne^{-n}=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{e^n}$,

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{(-1)^n(n+1)}{e^{n+1}}\cdot\frac{e^n}{(-1)^{n-1}n}\right|=\lim_{n\to\infty}\left|\frac{(-1)^1(n+1)}{en}\right|=\frac{1}{e}\lim_{n\to\infty}\frac{n+1}{n}=\frac{1}{e}(1)=\frac{1}{e}<1,$$

so the series is absolutely convergent (and therefore convergent). Now $b_6=6/e^6\approx 0.015>0.01$ and $b_7=7/e^7\approx 0.006<0.01$, so by the Alternating Series Estimation Theorem, $n=6$. (That is, since the 7th term is less than the desired error, we need to add the first 6 terms to get the sum to the desired accuracy.)

22. $\sum_{n=1}^{\infty} \left|(-1)^n \frac{n}{n^2+1}\right| = \sum_{n=1}^{\infty} \frac{n}{n^2+1} = \sum_{n=1}^{\infty} a_n$. If $b_n = \frac{1}{n}$, then $\sum_{n=1}^{\infty} b_n$ is the divergent harmonic series. Applying the Limit Comparison Test, $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n/(n^2+1)}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1 > 0$, so both series diverge and the given series is *not* absolutely convergent. (The Integral Test could also be used.)

26. $\left|\frac{\sin 4n}{4^n}\right| \le \frac{1}{4^n}$, so $\sum_{n=1}^{\infty} \left|\frac{\sin 4n}{4^n}\right|$ converges by comparison with the convergent geometric series $\sum_{n=1}^{\infty} \frac{1}{4^n}$ $\left(|r| = \frac{1}{4} < 1\right)$. Thus, $\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$ is absolutely convergent.