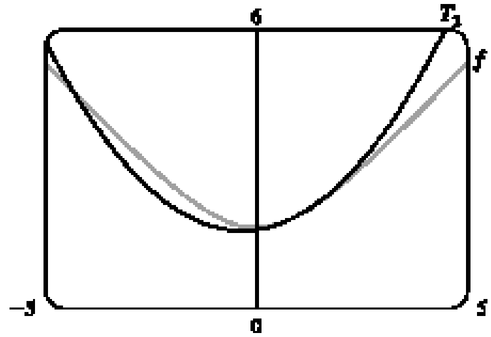
8.

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	$(3+x^2)^{1/2}$	2
1	$x(3+x^2)^{-1/2}$	$\frac{1}{2}$
2	$3(3+x^2)^{-3/2}$	$\frac{3}{8}$



$$T_2(x) = \sum_{n=0}^{2} \frac{f^{(n)}(1)}{n!}(x-1)^n = 2 + \tfrac{1}{2}(x-1) + \tfrac{3/8}{2}(x-1)^2 = 2 + \tfrac{1}{2}(x-1) + \tfrac{3}{16}(x-1)^2$$

15.

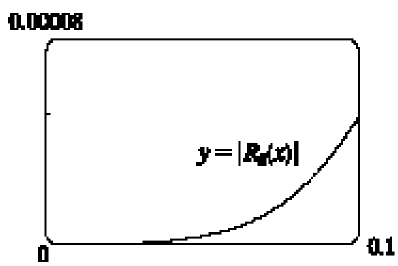
n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	e^{x^2}	1
1	$e^{x^2}(2x)$	0
2	$e^{x^2}(2+4x^2)$	2
3	$e^{x^2}(12x+8x^3)$	0
4	$e^{x^2}(12+48x^2+16x^4)$	

(a) $f(x) = e^{x^2} \approx T_3(x) = 1 + \frac{2}{2!}x^2 = 1 + x^2$

(b) $|R_3(x)| \le \dfrac{M}{4!}|x|^4$, where $\left|f^{(4)}(x)\right| \le M$. Now $0 \le x \le 0.1 \quad \Rightarrow \quad x^4 \le (0.1)^4$, and letting $x = 0.1$ gives

$$|R_3(x)| \le \frac{e^{0.01}(12 + 0.48 + 0.0016)}{24}(0.1)^4 \approx 0.00006.$$

(c)



From the graph of $|R_3(x)| = \left|e^{x^2} - (1+x^2)\right|$, it appears that the error is less than 0.000051 on $[0, 0.1]$.

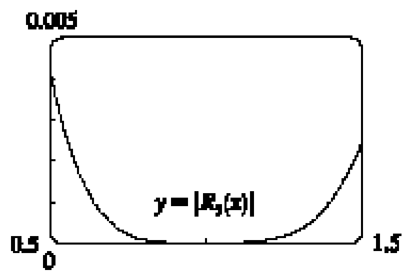
16.

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	$\ln(1+2x)$	$\ln 3$
1	$2/(1+2x)$	$\frac{2}{3}$
2	$-4/(1+2x)^2$	$-\frac{4}{9}$
3	$16/(1+2x)^3$	$\frac{16}{27}$
4	$-96/(1+2x)^4$	

(a) $f(x) = \ln(1+2x) \approx T_3(x) = \ln 3 + \frac{2}{3}(x-1) - \dfrac{4/9}{2!}(x-1)^2 + \dfrac{16/27}{3!}(x-1)^3$

(b) $|R_3(x)| \le \dfrac{M}{4!}|x-1|^4$, where $\left|f^{(4)}(x)\right| \le M$. Now $0.5 \le x \le 1.5 \Rightarrow -0.5 \le x-1 \le 0.5 \Rightarrow$ $|x-1| \le 0.5 \Rightarrow |x-1|^4 \le \frac{1}{16}$, and letting $x = 0.5$ gives $M = 6$, so $|R_3(x)| \le \dfrac{6}{4!} \cdot \dfrac{1}{16} = \dfrac{1}{64} = 0.015625$.

(c)



From the graph of $|R_3(x)| = |\ln(1+2x) - T_3(x)|$, it seems that the error is less than 0.005 on $[0.5, 1.5]$.

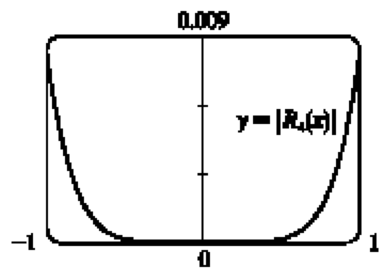
17.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$x \sin x$	0
1	$\sin x + x \cos x$	0
2	$2\cos x - x \sin x$	2
3	$-3\sin x - x\cos x$	0
4	$-4\cos x + x \sin x$	-4
5	$5\sin x + x\cos x$	

(a) $f(x) = x\sin x \approx T_4(x) = \frac{2}{2!}(x-0)^2 + \frac{-4}{4!}(x-0)^4 = x^2 - \frac{1}{6}x^4$

(b) $|R_4(x)| \le \frac{M}{5!}|x|^5$, where $\left|f^{(5)}(x)\right| \le M$. Now $-1 \le x \le 1 \;\Rightarrow\; |x| \le 1$, and a graph of $f^{(5)}(x)$ shows that $\left|f^{(5)}(x)\right| \le 5$ for $-1 \le x \le 1$. Thus, we can take $M = 5$ and get $|R_4(x)| \le \frac{5}{5!}\cdot 1^5 = \frac{1}{24} = 0.041\overline{6}$.

(c)



From the graph of $|R_4(x)| = |x\sin x - T_4(x)|$, it seems that the error is less than 0.0082 on $[-1, 1]$.

25. Let $s(t)$ be the position function of the car, and for convenience set $s(0) = 0$. The velocity of the car is $v(t) = s'(t)$ and the acceleration is $a(t) = s''(t)$, so the second degree Taylor polynomial is $T_2(t) = s(0) + v(0)t + \frac{a(0)}{2}t^2 = 20t + t^2$. We estimate the distance travelled during the next second to be $s(1) \approx T_2(1) = 20 + 1 = 21$ m. The function $T_2(t)$ would not be accurate over a full minute, since the car could not possibly maintain an acceleration of 2 m/s^2 for that long (if it did, its final speed would be 140 m/s $\approx$ 313 mi/h!)