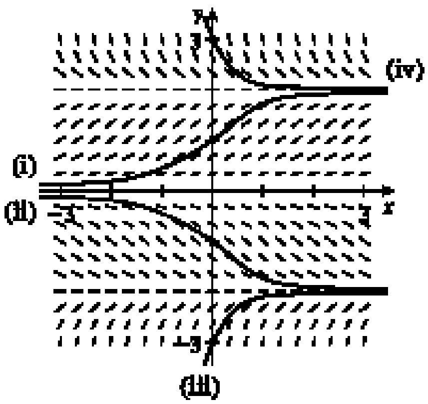
1. (a)



(b) It appears that the constant functions $y = 0$, $y = -2$, and $y = 2$ are equilibrium solutions. Note that these three values of y satisfy the given differential equation $y' = y\left(1 - \frac{1}{4}y^2\right)$.

3. $y' = y - 2$. The slopes at each point are independent of x, so the slopes are the same along each line parallel to the x-axis. Thus, III is the direction field for this equation. Note that for $y = 2$, $y' = 0$.

4. $y' = x(2 - y) = 0$ on the lines $x = 0$ and $y = 2$. Direction field I satisfies these conditions.

5. $y' = x + y - 1 = 0$ on the line $y = -x + 1$. Direction field IV satisfies this condition. Notice also that on the line $y = -x$ we have $y' = -1$, which is true in IV.

6. $y' = \sin x \sin y = 0$ on the lines $x = 0$ and $y = 0$, and $y' > 0$ for $0 < x < \pi$, $0 < y < \pi$. Direction field II satisfies these conditions.

Problem Set 23

Differential Equations Handout
Problems: 1, 2, 3, 4

April 26, 2006

Problem 1

Part A

We are given that

$$\frac{dx}{dt} = -2t$$

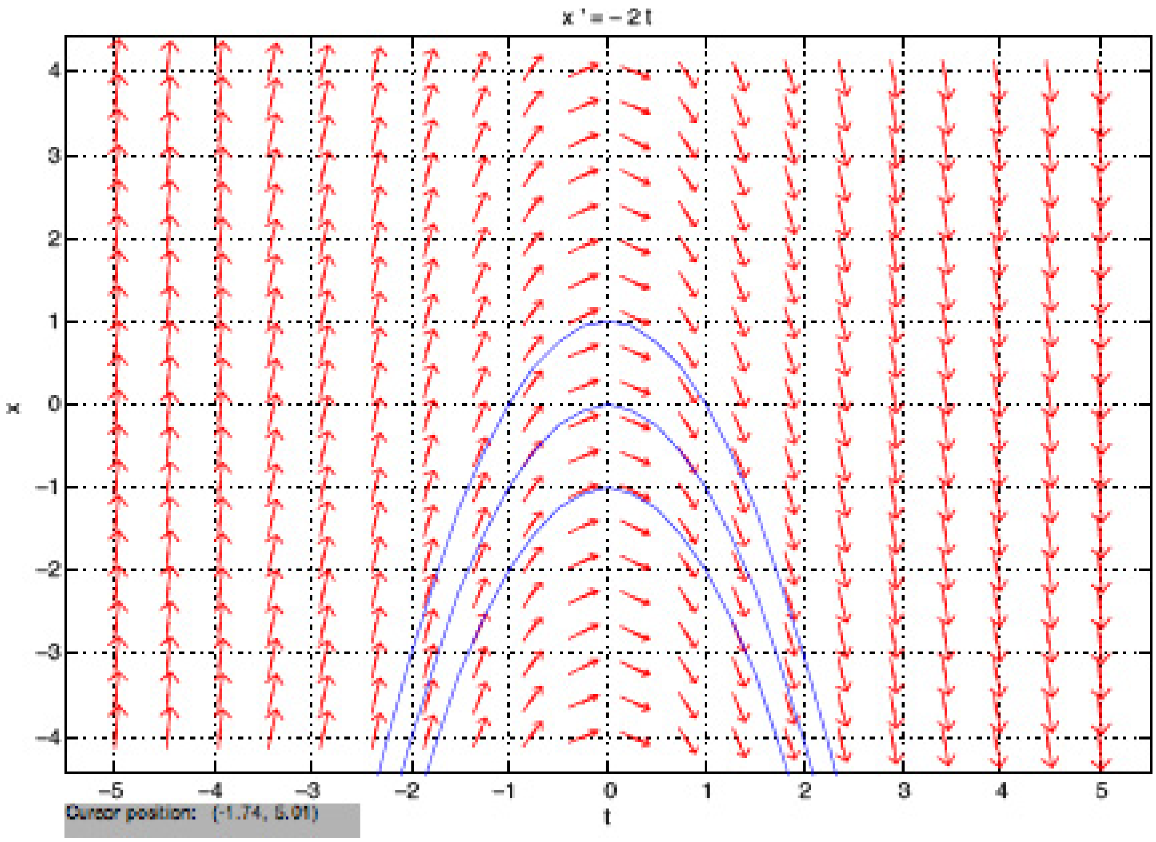
To solve this:

$$\frac{dx}{dt} = -2t \Rightarrow dx = -2tdt \Rightarrow \int dx = \int -2tdt \Rightarrow x = -t^2 + c$$

Thus our solutions is jut

$$x = -t^2 + c$$

where t must come from initial conditions. Given the condition that $x(0) = 1 \Rightarrow x = -t^2 + 1$. Given that $x(0) = 0 \Rightarrow x = -t^2$. Lastly, given that $x(0) = -1 \Rightarrow x = -t^2 - 1$. The slope field with the solution graphed for the three sets of initial conditions.



Part B

This follows similarly to part A. So we have that:

$$\frac{dx}{dt} = -2x \Rightarrow \frac{dx}{x} = -2dt \Rightarrow \int \frac{dx}{x} = \int -2dt \Rightarrow \ln x = -2t + C \Rightarrow x = e^{-2t+C} = ce^{-2t}$$

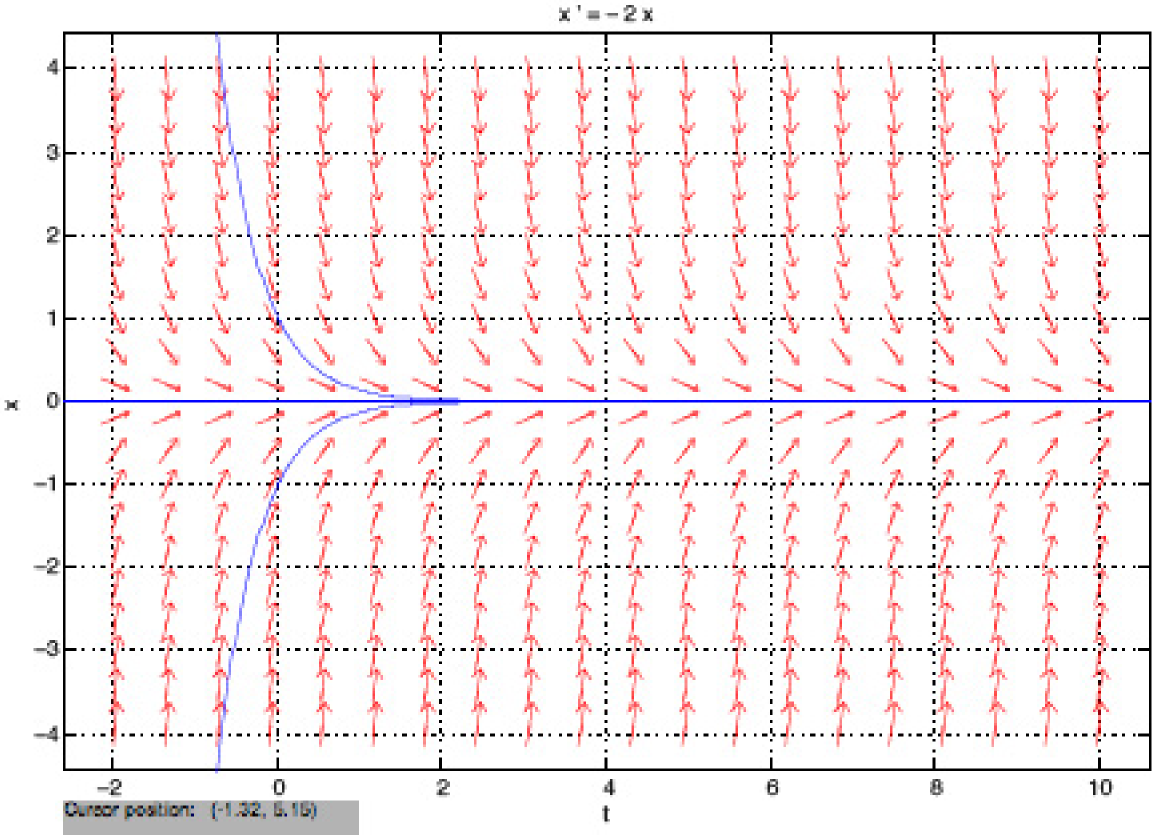
Given the three initial conditions, we get solutions of the form:

$$x(0) = 1 \Rightarrow x = e^{-2t}$$

$$x(0) = 0 \Rightarrow x = 0$$

$$x(0) = -1 \Rightarrow x = -e^{-2t}$$

The slope fields with the given initial conditions plotted are below:



Part C

So we have that:

$$\frac{dx}{dt} = t^2 \Rightarrow dx = t^2 dt \Rightarrow \int dx = \int t^2 dt \Rightarrow x = \frac{t^3}{3} + C$$

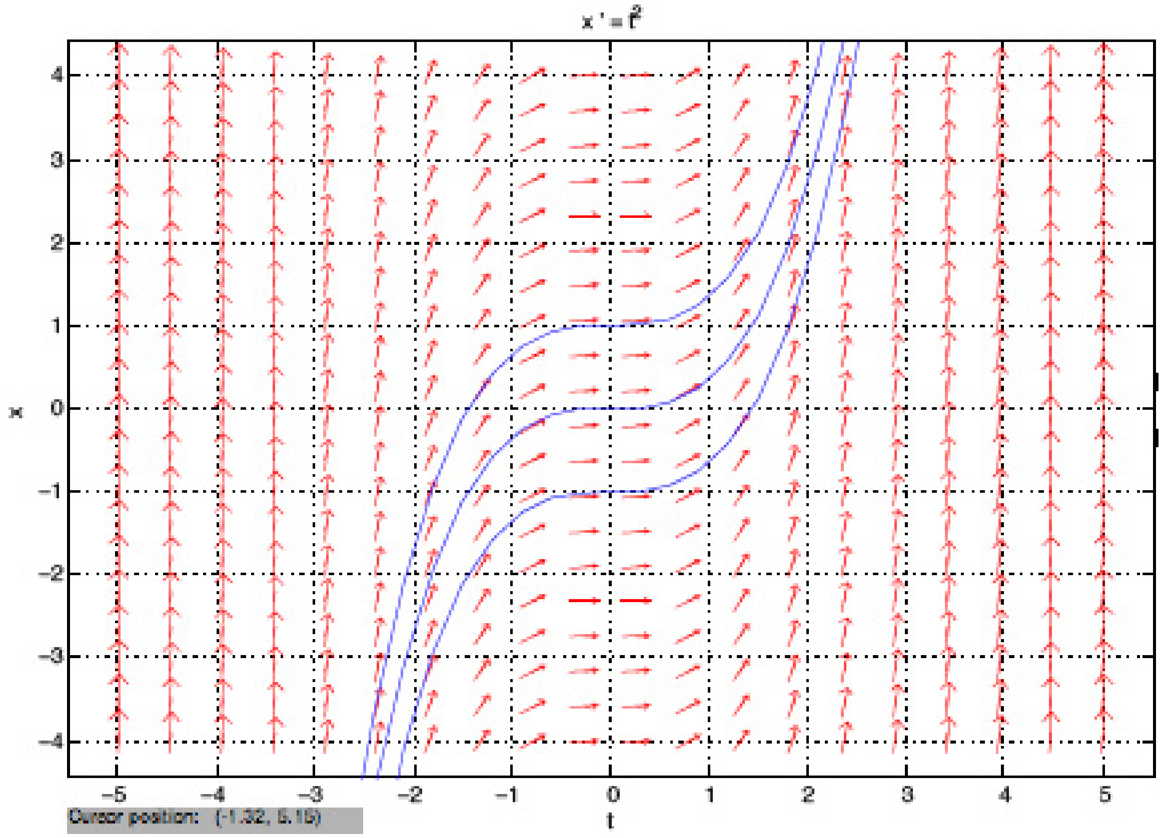
For the given initial conditions it will gives us that:

$$x(0) = 1 \Rightarrow x = \frac{t^3}{3} + 1$$

$$x(0) = 0 \Rightarrow x = \frac{t^3}{3}$$

$$x(0) = -1 \Rightarrow x = \frac{t^3}{3} - 1$$

The slope fields with the given initial conditions are plotted below.



Problem 2

We are given the differential equation

$$\frac{dy}{dt} = y + 1$$

Solution d will do exactly what we need. We can see this because:

$$y = Ce^t - 1 \Rightarrow \frac{dy}{dt} = Ce^t$$

Thus we have that

$$y + 1 = Ce^t - 1 + 1 = Ce^t = \frac{dy}{dt}$$

which is exactly what we need.

Problem 3

We are given the second order differential equation:

$$y'' + 9y = 0$$

Solution (e), $y = 5\cos 3t$ is exactly what we need since:

$$y = 5\cos 3t \Rightarrow y' = -15\sin 3t \Rightarrow y'' = -45\cos 3t$$

Thus we see that

$$y'' + 96 = -45\cos 3t + 9(5\cos 3t) = -45\cos 3t + 45\cos 3t = 0$$

Problem 4

It is given that the rate at which the concentration is changing is proportional to the difference in the concentration of the solute in the bloodstream (L) and that in the cell, $C(t)$. This we see that

$$\frac{dC(t)}{dt} = \alpha\,(L - C(t))$$

where $\alpha > 0$ since when there is more solute in the bloodstream, it will diffuse into the cell and $\frac{dC(t)}{dt}$ must be positive when $L > C(t)$.