# Problem Set 27

Differential Equation Handout - Exercises 13, 14, 16

May 8, 2006

Problem 13

Part A

The slope field is plotted below. As you can see from the given equations, there is a stable point at (0,0). There are also null lines along $y = 0$ and $x = 0$, as you can see since the arrows point perpendicular to the lines there. The solution with the initial point of $(4, 0)$ is also plotted (see Part B).
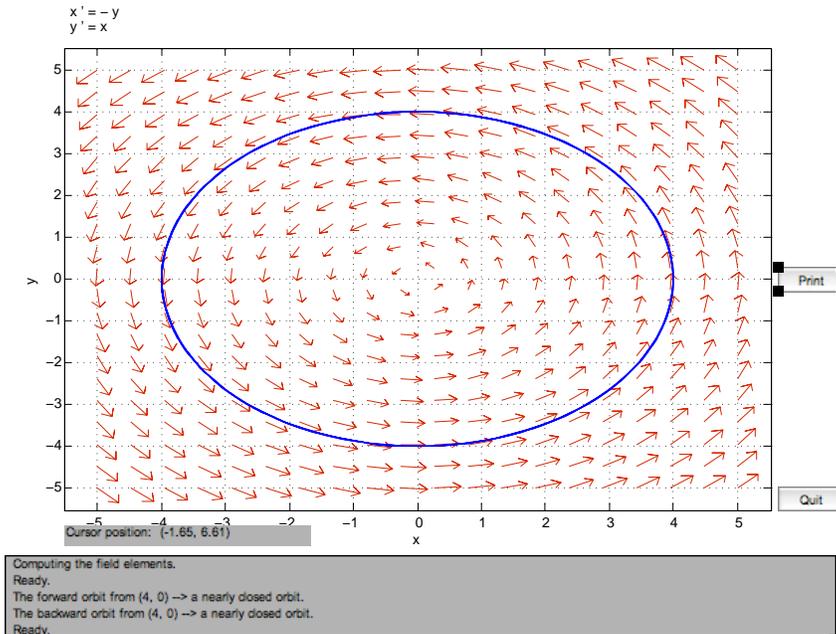
Part B

We are given the equations of

$$x = 4\cos t \qquad y = 4\sin t$$

This implies that

$$x' = -4\sin t \qquad y' = 4\cos t$$

As we can see this means that

$$x' = -y \qquad y' = x$$

which are exactly what we want. Also, we must verify that

$$x^2 + y^2 = (4\cos t)^2 + (4\sin t)^2 = 16\left(\cos^2 t + \sin^2 t\right) = 16$$

Problem 14

We are given the differential equations

$$x' = .1x - .05xy \qquad y' = .1y - .05xy$$

Part A

There are competitive because increase the number of one will decrease the rate of increase of the other.

Part B

These occur where x'=y'=0. This happens at $(x, y) = (0, 0)$ and $(x, y) = (2, 2)$

Part C

Null clines occur when on derivative is 0. If we set $x' = 0$ we get that $x = 0$ or $y = 2$. Plugging these solutions into y', we see that along the $x = 0$ null cline, $y' = .1y$ so the arrows will point away from the y-axis with increasing magnitude as you get farther away. Plugging in $y = 2$ into y' we find that $y' = .2 - .1x$ so you'll get arrows pointing to the left for $x > 2$ and to the right for $x < 2$. Similarly, setting $y' = 0$ we get that $y = 0$ or $x = 2$, with similar results for the directions of arrows. These are graphed below in the slope field (see next part).

Parts D-G

These parts each give us more information above the slope field. This is graphed below with the initial conditions from G superimposed.

x' = .1 x − .05 x y
y' = .1 y − .05 x y

Cursor position: [-1.65, 6.61]
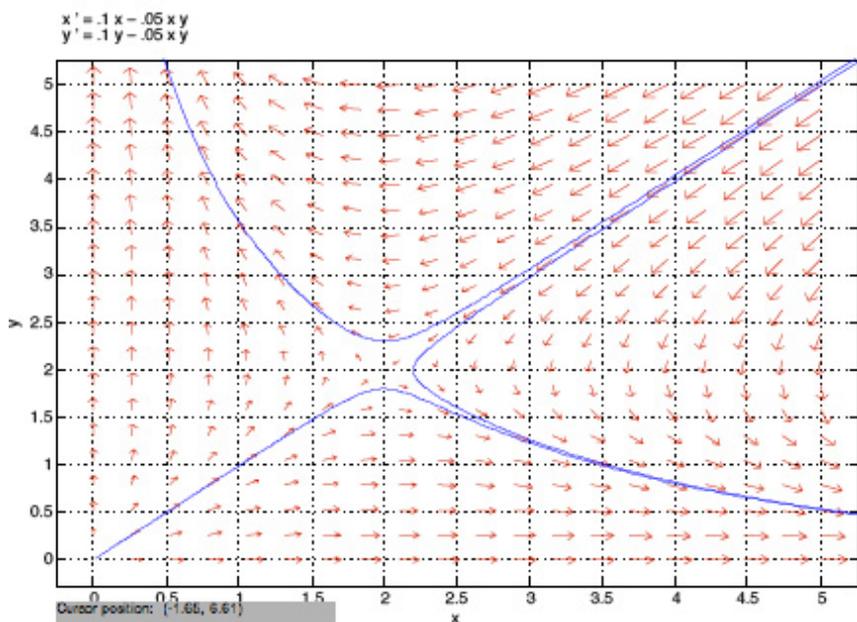
Part H

This does because there's only one equilibrium point where the species are in balance and both are alive, otherwise one dies out due to competition.

Problem 16

We are given the equations

$$x' = x - x^2 - axy \qquad y' = y - y^2 - axy$$

0.0.1 Part A

Let mice be x and rate be y. If we start out with no rats, then y=0 and y'=0. We well get that $x' = x - x^2$ so the population of mice will asymptotically approach $x = 1$ where we see that $x' = 0$. Similarly, if we start with no mice, then $x' = 0$ and we will asymptotically approach $y = 1$.

Part B

In general we will have equilibria points at $(x, y) = (0, 0), (1, 0), (0, 1), (\frac{1}{a+1}, \frac{1}{a+1})$. We will have null clines for $x' = 0$ along $x = 0$ and $y = \frac{1-x}{a}$. Similarly, we will have null clines for $y' = 0$ along $y = 0$ and $y = 1 - ax$. As we can see at $a = 1$, two of the null clines merge into one. Below are the null clines plotted, first $a = 1/2$, then $a = 3/2$.

$x' = x - x^2 - .5xy$
$y' = y - y^2 - .5xy$

$x' = x - x^2 - 1.5xy$
$y' = y - y^2 - 1.5xy$
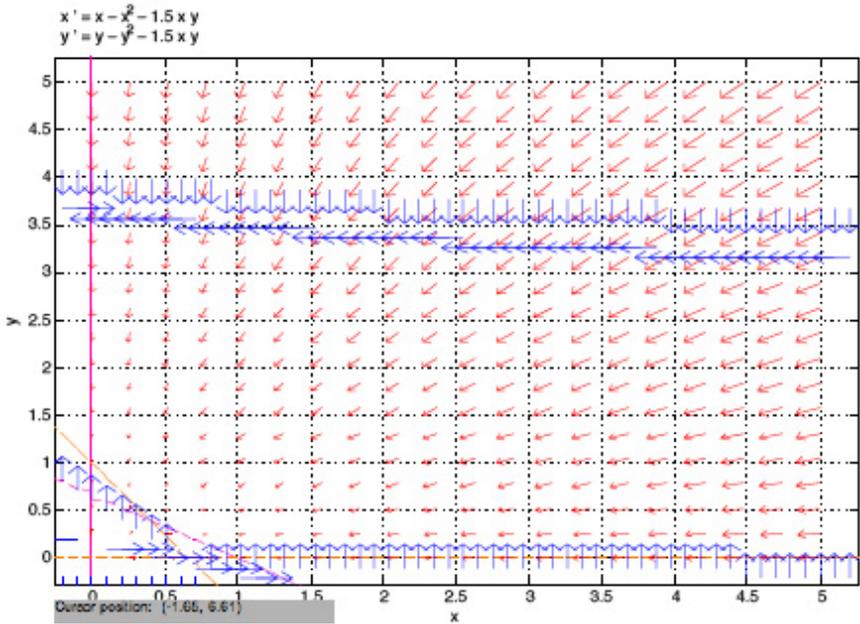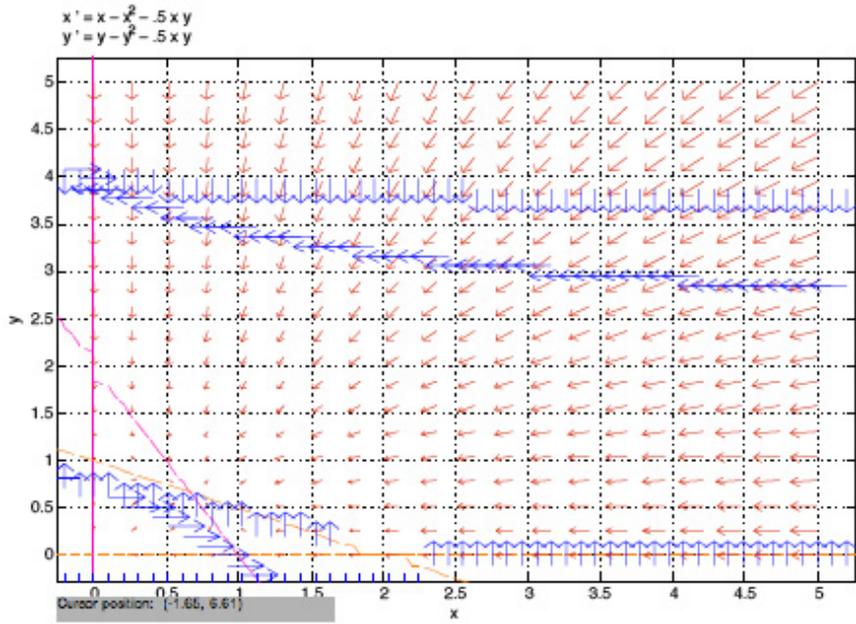
As we can see the purple and orange lines on the plot, which represent two of the null clines switch positions. They will be equal when $a = 1$. For $a < 1$ purple will always be steeper, which corresponds to the $x' = 0$ null cline being steeper. For $a > 1$ we see that $y = 1 - ax$ is steeper.