1. (a) See Section 6.1, Figure 2 and Equations 6.1.1 and 6.1.2.

(b) Instead of using "top minus bottom" and integrating from left to right, we use "right minus left" and integrate from bottom to top. See Figures 9 and 10 in Section 6.1.

2. The numerical value of the area represents the number of meters by which Sue is ahead of Kathy after 1 minute.

3. (a) See the discussion in Section 6.2, near Figures 2 and 3, ending in the Definition of Volume.

(b) See the discussion between Examples 5 and 6 in Section 6.2. If the cross-section is a disk, find the radius in terms of x or y and use $A = \pi(\text{radius})^2$. If the cross-section is a washer, find the inner radius r_{in} and outer radius r_{out} and use $A = \pi\left(r_{\text{out}}^2\right) - \pi\left(r_{\text{in}}^2\right)$.

4. (a) The length of a curve is defined to be the limit of the lengths of the inscribed polygons, as described near Figure 3 in Section 6.3.

(b) See Equation 6.3.1.

(c) See Equations 6.3.2 and 6.3.3.

5. (a) The average value of a function f on an interval $[a, b]$ is $f_{\text{ave}} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$.

(b) The Mean Value Theorem for Integrals says that there is a number c at which the value of f is exactly equal to the average value of the function, that is, $f(c) = f_{\text{ave}}$. For a geometric interpretation of the Mean Value Theorem for Integrals, see Figure 2 in Section 6.4 and the discussion that accompanies it.

6. $\int_0^6 f(x)\,dx$ represents the amount of work done. Its units are newton-meters, or joules.

7. Let $c(x)$ be the cross-sectional length of the wall (measured parallel to the surface of the fluid) at depth x. Then the hydrostatic force against the wall is given by $F = \int_a^b \delta x c(x)\,dx$, where a and b are the lower and upper limits for x at points of the wall and δ is the weight density of the fluid.

8. (a) The center of mass is the point at which the plate balances horizontally.

(b) See Equations 6.5.12.

9. See Figure 3 in Section 6.6, and the discussion which precedes it.

10. (a) See the definition in the first paragraph of the subsection *Cardiac Output* in Section 6.6.

(b) See the discussion in the second paragraph of the subsection *Cardiac Output* in Section 6.6.

11. A probability density function f is a function on the domain of a continuous random variable X such that $\int_a^b f(x)\,dx$ measures the probability that X lies between a and b. Such a function f has nonnegative values and satisfies the relation $\int_D f(x)\,dx = 1$, where D is the domain of the corresponding random variable X. If $D = \mathbb{R}$, or if we define $f(x) = 0$ for real numbers $x \notin D$, then $\int_{-\infty}^{\infty} f(x)\,dx = 1$. (Of course, to work with f in this way, we must assume that the integrals of f exist.)

12. (a) $\int_0^{130} f(x)\,dx$ represents the probability that the weight of a randomly chosen female college student is less than 130 pounds.

(b) $\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{\infty} x f(x)\,dx$

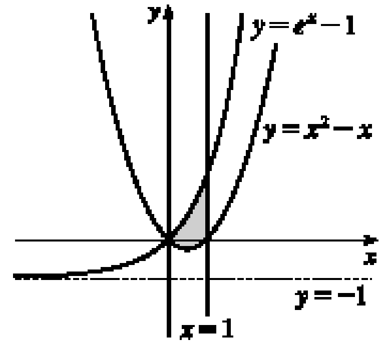
(c) The median of f is the number m such that $\int_m^{\infty} f(x)\,dx = \frac{1}{2}$.

13. See the discussion near Equation 3 in Section 6.7.

1. $A = \int_0^1 \left[(e^x - 1) - (x^2 - x)\right] dx$

$= \int_0^1 \left(e^x - 1 - x^2 + x\right) dx = \left[e^x - x - \frac{1}{3}x^3 + \frac{1}{2}x^2\right]_0^1$

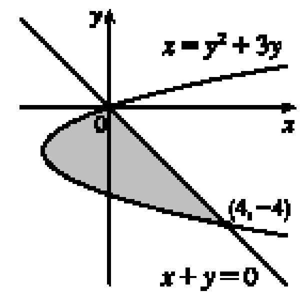
$= \left(e - 1 - \frac{1}{3} + \frac{1}{2}\right) - (1 - 0 - 0 + 0) = e - \frac{11}{6}$



2. $y^2 + 3y = -y \quad \Leftrightarrow \quad y^2 + 4y = 0 \quad \Leftrightarrow \quad y(y+4) = 0 \quad \Leftrightarrow$
$y = 0$ or -4.

$A = \int_{-4}^0 \left[-y - (y^2 + 3y)\right] dy = \int_{-4}^0 (-y^2 - 4y)\, dy$

$= \left[-\frac{1}{3}y^3 - 2y^2\right]_{-4}^0 = 0 - \left(\frac{64}{3} - 32\right) = \frac{32}{3}$

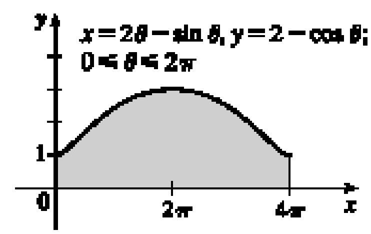


3. $x = 2\theta - \sin\theta \quad \Rightarrow \quad dx = (2 - \cos\theta)\, d\theta$

$A = \int_0^{2\pi} y\, dx = \int_0^{2\pi} \left[(2 - \cos\theta)(2 - \cos\theta)\right] d\theta$

$= \int_0^{2\pi} \left(4 - 4\cos\theta + \cos^2\theta\right) d\theta = \int_0^{2\pi} \left(4 - 4\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta$

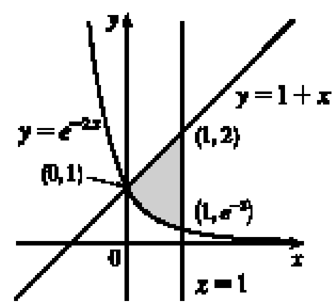
$= \left[4\theta - 4\sin\theta + \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{2\pi} = (8\pi - 0 + \pi + 0) - (0) = 9\pi$



4. $V = \int_0^1 \pi\left[(1+x)^2 - \left(e^{-2x}\right)^2\right] dx = \pi \int_0^1 \left(1 + 2x + x^2 - e^{-4x}\right) dx$

$= \pi\left[x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}e^{-4x}\right]_0^1 = \pi\left(1 + 1 + \frac{1}{3} + \frac{1}{4}e^{-4} - \frac{1}{4}\right)$

$= \pi\left(\dfrac{25}{12} + \dfrac{1}{4e^4}\right)$



5. (a) Using the Midpoint Rule on $[0, 1]$ with $f(x) = \tan\left(x^2\right)$ and $n = 4$, we estimate

$$A = \int_0^1 \tan\left(x^2\right) dx \approx \frac{1}{4}\left[\tan\left(\left(\tfrac{1}{8}\right)^2\right) + \tan\left(\left(\tfrac{3}{8}\right)^2\right) + \tan\left(\left(\tfrac{5}{8}\right)^2\right) + \tan\left(\left(\tfrac{7}{8}\right)^2\right)\right] \approx \frac{1}{4}(1.53) \approx 0.38$$

(b) Using the Midpoint Rule on $[0, 1]$ with $f(x) = \pi\tan^2\left(x^2\right)$ (for disks) and $n = 4$, we estimate

$$V = \int_0^1 f(x)\, dx \approx \frac{1}{4}\pi\left[\tan^2\left(\left(\tfrac{1}{8}\right)^2\right) + \tan^2\left(\left(\tfrac{3}{8}\right)^2\right) + \tan^2\left(\left(\tfrac{5}{8}\right)^2\right) + \tan^2\left(\left(\tfrac{7}{8}\right)^2\right)\right] \approx \frac{\pi}{4}(1.114) \approx 0.87$$

6. (a) $A = \int_0^1 (2x - x^2 - x^3)\,dx = \left[x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^1 = 1 - \frac{1}{3} - \frac{1}{4} = \frac{5}{12}$

(b) A cross-section is a washer with inner radius x^3 and outer radius $2x - x^2$, so its area is $\pi(2x - x^2)^2 - \pi(x^3)^2$.

$$V = \int_0^1 A(x)\,dx = \int_0^1 \pi[(2x - x^2)^2 - (x^3)^2]\,dx = \int_0^1 \pi(4x^2 - 4x^3 + x^4 - x^6)\,dx$$
$$= \pi\left[\tfrac{4}{3}x^3 - x^4 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7\right]_0^1 = \pi\left(\tfrac{4}{3} - 1 + \tfrac{1}{5} - \tfrac{1}{7}\right) = \tfrac{41\pi}{105}$$

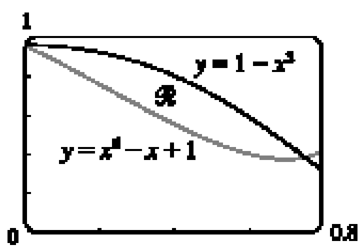
(c) Using the method of cylindrical shells,
$V = \int_0^1 2\pi x(2x - x^2 - x^3)\,dx = \int_0^1 2\pi(2x^2 - x^3 - x^4)\,dx = 2\pi\left[\frac{2}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5\right]_0^1 = 2\pi\left(\frac{2}{3} - \frac{1}{4} - \frac{1}{5}\right) = \frac{13\pi}{30}$.

7. (a) A cross-section is a washer with inner radius x^2 and outer radius x.
$V = \int_0^1 \pi\left[(x)^2 - (x^2)^2\right]dx = \int_0^1 \pi(x^2 - x^4)\,dx = \pi\left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_0^1 = \pi\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{2\pi}{15}$

(b) A cross-section is a washer with inner radius y and outer radius $\sqrt{y}$.
$V = \int_0^1 \pi\left[(\sqrt{y})^2 - y^2\right]dy = \int_0^1 \pi(y - y^2)\,dy = \pi\left[\frac{1}{2}y^2 - \frac{1}{3}y^3\right]_0^1 = \pi\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{\pi}{6}$

(c) A cross-section is a washer with inner radius $2 - x$ and outer radius $2 - x^2$.
$V = \int_0^1 \pi\left[(2 - x^2)^2 - (2 - x)^2\right]dx = \int_0^1 \pi(x^4 - 5x^2 + 4x)\,dx = \pi\left[\frac{1}{5}x^5 - \frac{5}{3}x^3 + 2x^2\right]_0^1 = \pi\left[\frac{1}{5} - \frac{5}{3} + 2\right] = \frac{8\pi}{15}$

8. (a)



From the graph, we see that the curves intersect at $x = 0$ and at $x = a \approx 0.75$, with $1 - x^2 > x^6 - x + 1$ on $(0, a)$.

(b) The area of $\mathcal{R}$ is $A = \int_0^a \left[(1 - x^2) - (x^6 - x + 1)\right]dx = \left[-\frac{1}{3}x^3 - \frac{1}{7}x^7 + \frac{1}{2}x^2\right]_0^a \approx 0.12$.

(c) Using washers, the volume generated when $\mathcal{R}$ is rotated about the x-axis is

$$V = \pi\int_0^a [(1 - x^2)^2 - (x^6 - x + 1)^2]\,dx = \pi\int_0^a (-x^{12} + 2x^7 - 2x^6 + x^4 - 3x^2 + 2x)\,dx$$
$$= \pi\left[-\tfrac{1}{13}x^{13} + \tfrac{1}{4}x^8 - \tfrac{2}{7}x^7 + \tfrac{1}{5}x^5 - x^3 + x^2\right]_0^a \approx 0.54$$

(d) Using shells, the volume generated when $\mathcal{R}$ is rotated about the y-axis is
$V = \int_0^a 2\pi x[(1 - x^2) - (x^6 - x + 1)]\,dx = 2\pi\int_0^a (-x^3 - x^7 + x^2)\,dx = 2\pi\left[-\frac{1}{4}x^4 - \frac{1}{8}x^8 + \frac{1}{3}x^3\right]_0^a \approx 0.31$.

9. (a) The solid is obtained by rotating the region $\mathcal{R} = \left\{(x, y) \mid 0 \le x \le \frac{\pi}{2}, 0 \le y \le \sqrt{2}\cos x\right\}$ about the x-axis.

(b) The solid is obtained by rotating the region $\mathcal{R} = \left\{(x, y) \mid 0 \le x \le 1, 2 - \sqrt{x} \le y \le 2 - x^2\right\}$ about the x-axis.
Or: The solid is obtained by rotating the region $\mathcal{R} = \left\{(x, y) \mid 0 \le x \le 1, x^2 \le y \le \sqrt{x}\right\}$ about the line $y = 2$.

10. With an x-axis in the normal position, at $x = 7$ we have $C = 2\pi r = 45 \quad \Rightarrow \quad r(7) = \frac{2\pi}{45}$.
Using Simpson's Rule with $n = 4$ and $\Delta x = 7$, we have
$V = \int_0^{28} \pi[r(x)]^2 \, dx \approx S_4 = \frac{7}{3}\left[0 + 4\pi\left(\frac{45}{2\pi}\right)^2 + 2\pi\left(\frac{53}{2\pi}\right)^2 + 4\pi\left(\frac{45}{2\pi}\right)^2 + 0\right] = \frac{7}{3}\left(\frac{21{,}818}{4\pi}\right) \approx 4051 \text{ cm}^3$.

11. Take the base to be the disk $x^2 + y^2 \leq 9$. Then $V = \int_{-3}^{3} A(x)\, dx$, where $A(x_0)$ is the area of the isosceles right triangle whose hypotenuse lies along the line $x = x_0$ in the xy-plane. The length of the hypotenuse is $2\sqrt{9 - x^2}$ and the length of each leg is $\sqrt{2}\sqrt{9 - x^2}$. $A(x) = \frac{1}{2}\left(\sqrt{2}\sqrt{9 - x^2}\right)^2 = 9 - x^2$, so

$$V = 2\int_0^3 A(x)\, dx = 2\int_0^3 (9 - x^2)\, dx = 2\left[9x - \tfrac{1}{3}x^3\right]_0^3 = 2(27 - 9) = 36$$

12. $V = \int_{-1}^{1} A(x)\, dx = 2\int_0^1 A(x)\, dx = 2\int_0^1 \left[(2 - x^2) - x^2\right]^2 dx = 2\int_0^1 \left[2(1 - x^2)\right]^2 dx$
$= 8\int_0^1 (1 - 2x^2 + x^4)\, dx = 8\left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]_0^1 = 8\left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{64}{15}$

13. Equilateral triangles with sides measuring $\frac{1}{4}x$ meters have height $\frac{1}{4}x \sin 60^\circ = \frac{\sqrt{3}}{8}x$. Therefore,
$A(x) = \frac{1}{2} \cdot \frac{1}{4}x \cdot \frac{\sqrt{3}}{8}x = \frac{\sqrt{3}}{64}x^2$. $V = \int_0^{20} A(x)\, dx = \frac{\sqrt{3}}{64}\int_0^{20} x^2\, dx = \frac{\sqrt{3}}{64}\left[\frac{1}{3}x^3\right]_0^{20} = \frac{8000\sqrt{3}}{64 \cdot 3} = \frac{125\sqrt{3}}{3} \text{ m}^3$.

14. (a) By the symmetry of the problem, we consider only the solid to the right of the origin. The semicircular cross-sections perpendicular to the x-axis have radius $1 - x$, so $A(x) = \frac{1}{2}\pi(1 - x)^2$. Now we can calculate
$V = 2\int_0^1 A(x)\, dx = 2\int_0^1 \frac{1}{2}\pi(1 - x)^2\, dx = \int_0^1 \pi(1 - x)^2\, dx = -\frac{\pi}{3}\left[(1 - x)^3\right]_0^1 = \frac{\pi}{3}$.

(b) Cut the solid with a plane perpendicular to the x-axis and passing through the y-axis. Fold the half of the solid in the region $x \leq 0$ under the xy-plane so that the point $(-1, 0)$ comes around and touches the point $(1, 0)$. The resulting solid is a right circular cone of radius 1 with vertex at $(x, y, z) = (1, 0, 0)$ and with its base in the yz-plane, centered at the origin. The volume of this cone is $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 1^2 \cdot 1 = \frac{\pi}{3}$.

15. $x = 3t^2, y = 2t^3, 0 \leq t \leq 2$.
$L = \int_0^2 \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt = \int_0^2 \sqrt{(6t)^2 + (6t^2)^2}\, dt = \int_0^2 \sqrt{36t^2 + 36t^4}\, dt = 6\int_0^2 t\sqrt{1 + t^2}\, dt$
$= 6\int_1^5 \sqrt{u}\left(\frac{1}{2}du\right) \quad \left[u = 1 + t^2, du = 2t\, dt\right] \quad = 3\left[\frac{2}{3}u^{3/2}\right]_1^5 = 2(5\sqrt{5} - 1)$

16. $y = \dfrac{1}{x^2}, 1 \leq x \leq 2$. $\dfrac{dy}{dx} = -\dfrac{2}{x^3}$, so $1 + \left(\dfrac{dy}{dx}\right)^2 = 1 + \dfrac{4}{x^6} \quad \Rightarrow \quad f(x) = \sqrt{1 + 4/x^6}$ and $L = \int_1^2 \sqrt{1 + 4/x^6}\, dx$. By Simpson's Rule with $n = 10$, $L \approx \frac{1/10}{3}[f(1) + 4f(1.1) + 2f(1.2) + 4f(1.3) + \cdots + 2f(1.8) + 4f(1.9) + f(2)] \approx 1.297$.

17. $y = \frac{1}{6}(x^2+4)^{3/2} \Rightarrow dy/dx = \frac{1}{4}(x^2+4)^{1/2}(2x) \Rightarrow$

$1 + (dy/dx)^2 = 1 + \left[\frac{1}{2}x(x^2+4)^{1/2}\right]^2 = 1 + \frac{1}{4}x^2(x^2+4) = \frac{1}{4}x^4 + x^2 + 1 = \left(\frac{1}{2}x^2+1\right)^2.$

Thus, $L = \int_0^3 \sqrt{\left(\frac{1}{2}x^2+1\right)^2}\,dx = \int_0^3 \left(\frac{1}{2}x^2+1\right)dx = \left[\frac{1}{6}x^3 + x\right]_0^3 = \frac{15}{2}.$

18. $y = \int_1^x \sqrt{\sqrt{t}-1}\,dt \Rightarrow dy/dx = \sqrt{\sqrt{x}-1} \Rightarrow 1 + (dy/dx)^2 = 1 + (\sqrt{x}-1) = \sqrt{x}.$

Thus, $L = \int_1^{16} \sqrt{\sqrt{x}}\,dx = \int_1^{16} x^{1/4}\,dx = \frac{4}{5}\left[x^{5/4}\right]_1^{16} = \frac{4}{5}(32-1) = \frac{124}{5}.$

19. $f(x) = kx \Rightarrow 30\text{ N} = k(15-12)\text{ cm} \Rightarrow k = 10\text{ N/cm} = 1000\text{ N/m}.\quad 20\text{ cm} - 12\text{ cm} = 0.08\text{ m} \Rightarrow$

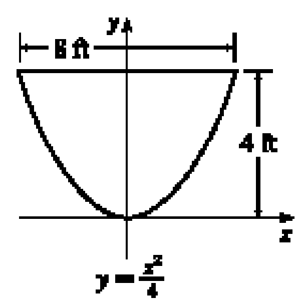
$W = \int_0^{0.08} kx\,dx = 1000\int_0^{0.08} x\,dx = 500\left[x^2\right]_0^{0.08} = 500(0.08)^2 = 3.2\text{ N-m} = 3.2\text{ J}.$

20. The work needed to raise the elevator alone is 1600 lb $\times$ 30 ft $=$ 48,000 ft-lb. The work needed to raise the bottom 170 ft of cable is 170 ft $\times$ 10 lb/ft $\times$ 30 ft $=$ 51,000 ft-lb. The work needed to raise the top 30 ft of cable is $\int_0^{30} 10x\,dx = \left[5x^2\right]_0^{30} = 5\cdot 900 = 4500$ ft-lb. Adding these, we see that the total work needed is $48{,}000 + 51{,}000 + 4{,}500 = 103{,}500$ ft-lb.

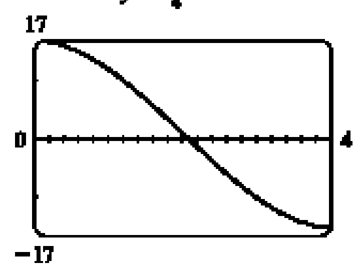
21. (a) The parabola has equation $y = ax^2$ with vertex at the origin and passing through $(4,4)$. $4 = a\cdot 4^2 \Rightarrow a = \frac{1}{4} \Rightarrow y = \frac{1}{4}x^2 \Rightarrow x^2 = 4y \Rightarrow x = 2\sqrt{y}$. Each circular disk has radius $2\sqrt{y}$ and is moved $4-y$ ft.

$$W = \int_0^4 \pi\left(2\sqrt{y}\right)^2 62.5\,(4-y)\,dy = 250\pi\int_0^4 y(4-y)\,dy$$

$$= 250\pi\left[2y^2 - \frac{1}{3}y^3\right]_0^4 = 250\pi\left(32 - \frac{64}{3}\right) = \frac{8000\pi}{3} \approx 8378\text{ ft-lb}$$



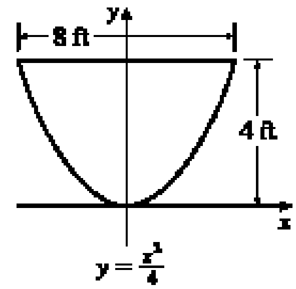
(b) In part (a) we knew the final water level (0) but not the amount of work done. Here we use the same equation, except with the work fixed, and the lower limit of integration (that is, the final water level — call it h) unknown: $W = 4000 \Leftrightarrow$ $250\pi\left[2y^2 - \frac{1}{3}y^3\right]_h^4 = 4000 \Leftrightarrow \frac{16}{\pi} = \left[\left(32 - \frac{64}{3}\right) - \left(2h^2 - \frac{1}{3}h^3\right)\right] \Leftrightarrow$ $h^3 - 6h^2 + 32 - \frac{48}{\pi} = 0$. We graph the function $f(h) = h^3 - 6h^2 + 32 - \frac{48}{\pi}$ on the interval $[0,4]$ to see where it is 0. From the graph, $f(h) = 0$ for $h \approx 2.1$. So the depth of water remaining is about 2.1 ft.



22. $F = \int_0^4 \delta(4-y)2\left(2\sqrt{y}\right)dy = 4\delta\int_0^4\left(4y^{1/2} - y^{3/2}\right)dy$

$= 4\delta\left[\frac{8}{3}y^{3/2} - \frac{2}{5}y^{5/2}\right]_0^4 = 4\delta\left(\frac{64}{3} - \frac{64}{5}\right) = 256\delta\left(\frac{1}{3} - \frac{1}{5}\right)$

$= \frac{512}{15}\delta \approx 2133.3 \text{ lb} \qquad [\delta \approx 62.5 \text{ lb/ft}^3]$



23. As in Example 4 of Section 6.5, $\dfrac{a}{2-x} = \dfrac{1}{2} \quad\Rightarrow\quad 2a = 2 - x$ and $w = 2(1.5 + a) = 3 + 2a = 3 + 2 - x = 5 - x$.

Thus, $F = \int_0^2 \rho g x(5-x)\,dx = \rho g\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^2 = \rho g\left(10 - \frac{8}{3}\right) = \frac{22}{3}\delta \;\; [\rho g = \delta] \;\; \approx \frac{22}{3}\cdot 62.5 \approx 458$ lb.

24. An equation of the line passing through $(0,0)$ and $(3,2)$ is $y = \frac{2}{3}x$. $A = \frac{1}{2}\cdot 3\cdot 2 = 3$. Therefore, using Equations 6.5.12, $\overline{x} = \frac{1}{3}\int_0^3 x\left(\frac{2}{3}x\right)dx = \frac{2}{27}\left[x^3\right]_0^3 = 2$ and $\overline{y} = \frac{1}{3}\int_0^3 \frac{1}{2}\left(\frac{2}{3}x\right)^2 dx = \frac{2}{81}\left[x^3\right]_0^3 = \frac{2}{3}$. Thus, the centroid is $(\overline{x}, \overline{y}) = \left(2, \frac{2}{3}\right)$.

25. $x = 100 \quad\Rightarrow\quad P = 2000 - 0.1(100) - 0.01(100)^2 = 1890$

$$\text{Consumer surplus} = \int_0^{100}[p(x) - P]\,dx = \int_0^{100}\left(2000 - 0.1x - 0.01x^2 - 1890\right)dx$$

$$= \left[110x - 0.05x^2 - \tfrac{0.01}{3}x^3\right]_0^{100} = 11{,}000 - 500 - \tfrac{10{,}000}{3} \approx \$7166.67$$

26. $f_{\text{ave}} = \frac{1}{2-0}\int_0^2 x^2\sqrt{1+x^3}\,dx = \frac{1}{2}\cdot\frac{1}{3}\int_1^9 \sqrt{u}\,du \qquad [u = 1 + x^3,\, du = 3x^2\,dx]$

$= \frac{1}{6}\left[\frac{2}{3}u^{3/2}\right]_1^9 = (9^{3/2} - 1^{3/2}) = \frac{1}{9}(27 - 1) = \frac{26}{9}$

27. $\displaystyle\lim_{h\to 0} f_{\text{ave}} = \lim_{h\to 0}\frac{1}{(x+h) - x}\int_x^{x+h} f(t)\,dt = \lim_{h\to 0}\frac{F(x+h) - F(x)}{h}$, where $F(x) = \int_a^x f(t)\,dt$. But we recognize this limit as being $F'(x)$ by the definition of a derivative. Therefore, $\lim_{h\to 0} f_{\text{ave}} = F'(x) = f(x)$ by FTC1.

28. $\int_0^{24} c(t)\,dt \approx S_{12} = \frac{24-0}{12\cdot 3}[1(0) + 4(1.9) + 2(3.3) + 4(5.1) + 2(7.6) + 4(7.1) + 2(5.8)$

$+ 4(4.7) + 2(3.3) + 4(2.1) + 2(1.1) + 4(0.5) + 1(0)]$

$= \frac{2}{3}(127.8) = 85.2 \text{ mg}\cdot\text{s/L}$

Therefore, $F \approx A/85.2 = 6/85.2 \approx 0.0704$ L/s or 4.225 L/min.

29. $f(x) = \begin{cases} \frac{\pi}{20}\sin\left(\frac{\pi}{10}x\right) & \text{if } 0 \le x \le 10 \\ 0 & \text{if } x < 0 \text{ or } x > 10 \end{cases}$

(a) $f(x) \ge 0$ for all real numbers x and

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{10} \frac{\pi}{20}\sin\left(\frac{\pi}{10}x\right)dx = \frac{\pi}{20}\cdot\frac{10}{\pi}\left[-\cos\left(\frac{\pi}{10}x\right)\right]_0^{10} = \frac{1}{2}(-\cos\pi + \cos 0) = \frac{1}{2}(1+1) = 1$$

Therefore, f is a probability density function.

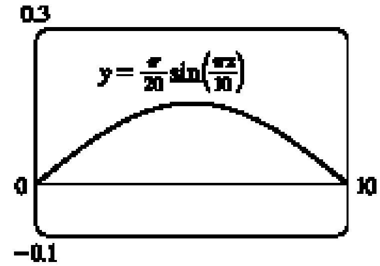
(b) $P(X < 4) = \int_{-\infty}^{4} f(x)\,dx = \int_0^4 \frac{\pi}{20}\sin\left(\frac{\pi}{10}x\right)dx = \frac{1}{2}\left[-\cos\left(\frac{\pi}{10}x\right)\right]_0^4 = \frac{1}{2}\left(-\cos\frac{2\pi}{5} + \cos 0\right)$

$\approx \frac{1}{2}(-0.309017 + 1) \approx 0.3455$

(c) $\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{10} \frac{\pi}{20} x \sin\left(\frac{\pi}{10}x\right)dx$

$= \int_0^{\pi} \frac{\pi}{20}\cdot\frac{10}{\pi}u(\sin u)\left(\frac{10}{\pi}\right)du \quad \left[u = \frac{\pi}{10}x,\ du = \frac{\pi}{10}\,dx\right]$

$= \frac{5}{\pi}\int_0^{\pi} u \sin u\,du \overset{82}{=} \frac{5}{\pi}\left[\sin u - u\cos u\right]_0^{\pi} = \frac{5}{\pi}[0 - \pi(-1)] = 5$



This answer is expected because the graph of f is symmetric about the line $x = 5$.

30. $P(250 \le X \le 280) = \displaystyle\int_{250}^{280} \frac{1}{15\sqrt{2\pi}} \exp\left(\frac{-(x-268)^2}{2\cdot 15^2}\right) dx \approx 0.673$. Thus, the percentage of pregnancies that last between 250 and 280 days is about 67.3%.

31. (a) The probability density function is $f(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{8}e^{-t/8} & \text{if } t \ge 0 \end{cases}$

$P(0 \le X \le 3) = \int_0^3 \frac{1}{8}e^{-t/8}\,dt = \left[-e^{-t/8}\right]_0^3 = -e^{-3/8} + 1 \approx 0.3127$

(b) $P(X > 10) = \int_{10}^{\infty} \frac{1}{8}e^{-t/8}\,dt = \lim\limits_{x\to\infty}\left[-e^{-t/8}\right]_{10}^{x} = \lim\limits_{x\to\infty}\left(-e^{-x/8} + e^{-10/8}\right) = 0 + e^{-5/4} \approx 0.2865$

(c) We need to find m such that $P(X \ge m) = \frac{1}{2} \Rightarrow \int_m^{\infty} \frac{1}{8}e^{-t/8}\,dt = \frac{1}{2} \Rightarrow \lim\limits_{x\to\infty}\left[-e^{-t/8}\right]_m^x = \frac{1}{2} \Rightarrow$

$\lim\limits_{x\to\infty}\left(-e^{-x/8} + e^{-m/8}\right) = \frac{1}{2} \Rightarrow e^{-m/8} = \frac{1}{2} \Rightarrow -m/8 = \ln\frac{1}{2} \Rightarrow m = -8\ln\frac{1}{2} = 8\ln 2 \approx 5.55$ minutes.