1. (a) A differential equation is an equation that contains an unknown function and one or more of its derivatives.

(b) The order of a differential equation is the order of the highest derivative that occurs in the equation.

(c) An initial condition is a condition of the form $y(t_0) = y_0$.

2. $y' = x^2 + y^2 \geq 0$ for all x and y. $y' = 0$ only at the origin, so there is a horizontal tangent at $(0, 0)$, but nowhere else. The graph of the solution is increasing on every interval.

3. See the paragraph preceding Example 1 in Section 7.2.

4. See the paragraph after Figure 14 in Section 7.2.

5. A separable equation is a first-order differential equation in which the expression for dy/dx can be factored as a function of x times a function of y, that is, $dy/dx = g(x)f(y)$. We can solve the equation by integrating both sides of the equation $dy/f(y) = g(x)dx$ and solving for y.

6. (a) $\dfrac{dy}{dt} = ky$; the relative growth rate, $\dfrac{1}{y}\dfrac{dy}{dt}$, is constant.

(b) The equation in part (a) is an appropriate model for population growth, assuming that there is enough room and nutrition to support the growth.

(c) If $y(0) = y_0$, then the solution is $y(t) = y_0e^{kt}$.

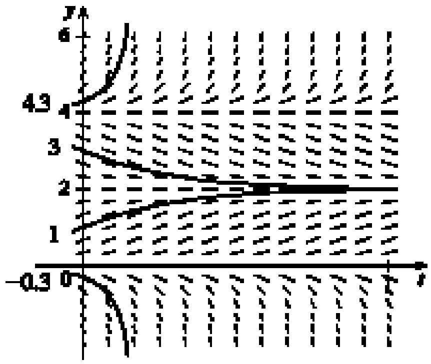
7. (a) $dP/dt = kP(1 - P/K)$, where K is the carrying capacity.

(b) The equation in part (a) is an appropriate model for population growth, assuming that the population grows at a rate proportional to the size of the population in the beginning, but eventually levels off and approaches its carrying capacity because of limited resources.

8. (a) $dF/dt = kF - aFS$ and $dS/dt = -rS + bFS$.

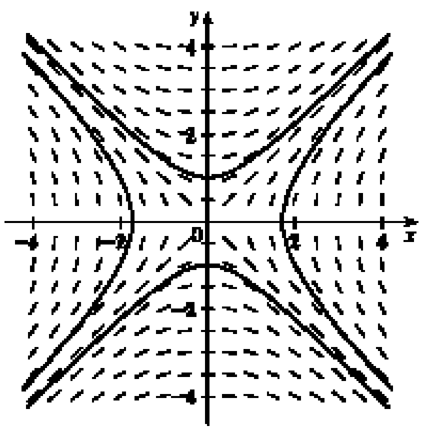
(b) In the absence of sharks, an ample food supply would support exponential growth of the fish population, that is, $dF/dt = kF$, where k is a positive constant. In the absence of fish, we assume that the shark population would decline at a rate proportional to itself, that is, $dS/dt = -rS$, where r is a positive constant.

1. (a)



(b) $\lim_{t\to\infty} y(t)$ appears to be finite for $0 \le c \le 4$. In fact $\lim_{t\to\infty} y(t) = 4$ for $c = 4$, $\lim_{t\to\infty} y(t) = 2$ for $0 < c < 4$, and $\lim_{t\to\infty} y(t) = 0$ for $c = 0$. The equilibrium solutions are $y(t) = 0$, $y(t) = 2$, and $y(t) = 4$.

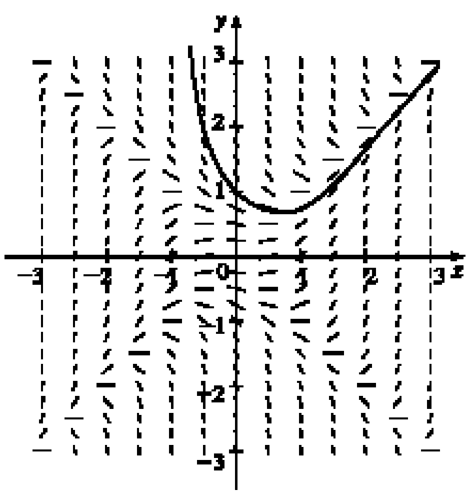
2. (a)



We sketch the direction field and four solution curves, as shown. Note that the slope $y' = x/y$ is not defined on the line $y = 0$.

(b) $y' = x/y \quad\Leftrightarrow\quad y\,dy = x\,dx \quad\Leftrightarrow\quad y^2 = x^2 + C$. For $C = 0$, this is the pair of lines $y = \pm x$. For $C \neq 0$, it is the hyperbola $x^2 - y^2 = -C$.

3. (a)



(b) $h = 0.1$, $x_0 = 0$, $y_0 = 1$ and $F(x, y) = x^2 - y^2$.

So $y_n = y_{n-1} + 0.1\left(x_{n-1}^2 - y_{n-1}^2\right)$. Thus,

$y_1 = 1 + 0.1\left(0^2 - 1^2\right) = 0.9$,

$y_2 = 0.9 + 0.1\left(0.1^2 - 0.9^2\right) = 0.82$,

$y_3 = 0.82 + 0.1\left(0.2^2 - 0.82^2\right) = 0.75676$. This is close to our graphical estimate of $y(0.3) \approx 0.8$.

We estimate that when $x = 0.3$, $y = 0.8$, so $y(0.3) \approx 0.8$.

(c) The centers of the horizontal line segments of the direction field are located on the lines $y = x$ and $y = -x$. When a solution curve crosses one of these lines, it has a local maximum or minimum.

4. (a) $h = 0.2$, $x_0 = 0$, $y_0 = 1$ and $F(x, y) = 2xy^2$. We need y_2.

$y_1 = 1 + 0.2\big(2 \cdot 0 \cdot 1^2\big) = 1$, $y_2 = 1 + 0.2\big(2 \cdot 0.2 \cdot 1^2\big) = 1.08 \approx y(0.4)$.

(b) $h = 0.1$ now, so $y_1 = 1 + 0.1\big(2 \cdot 0 \cdot 1^2\big) = 1$, $y_2 = 1 + 0.1\big(2 \cdot 0.1 \cdot 1^2\big) = 1.02$,

$y_3 = 1.02 + 0.1\big(2 \cdot 0.2 \cdot 1.02^2\big) \approx 1.06162$, $y_4 = 1.06162 + 0.1\big(2 \cdot 0.3 \cdot 1.06162^2\big) \approx 1.1292 \approx y(0.4)$.

(c) The equation is separable, so we write $\dfrac{dy}{y^2} = 2x\,dx \;\Rightarrow\; \displaystyle\int \frac{dy}{y^2} = \int 2x\,dx \;\Leftrightarrow\; -\frac{1}{y} = x^2 + C$, but $y(0) = 1$, so $C = -1$ and $y(x) = \dfrac{1}{1 - x^2} \;\Leftrightarrow\; y(0.4) = \dfrac{1}{1 - 0.16} \approx 1.1905$. From this we see that the approximation was greatly improved by increasing the number of steps, but the approximations were still far off.

5. $(3y^2 + 2y)\,y' = x\cos x \;\Rightarrow\; (3y^2 + 2y)\,dy = (x\cos x)\,dx \;\Rightarrow\; \int(3y^2 + 2y)\,dy = \int(x\cos x)\,dx \;\Rightarrow$
$y^3 + y^2 = \cos x + x\sin x + C$. For the last step, use integration by parts or Formula 83 in the Table of Integrals.

6. $\dfrac{dx}{dt} = 1 - t + x - tx = 1(1 - t) + x(1 - t) = (1 + x)(1 - t) \;\Rightarrow\; \dfrac{dx}{1 + x} = (1 - t)\,dt \;\Rightarrow$
$\displaystyle\int \frac{dx}{1 + x} = \int (1 - t)\,dt \;\Rightarrow\; \ln|1 + x| = t - \tfrac{1}{2}t^2 + C \;\Rightarrow\; |1 + x| = e^{t - t^2/2 + C} \;\Rightarrow$
$1 + x = \pm e^{t - t^2/2} \cdot e^C \;\Rightarrow\; x = -1 + Ke^{t - t^2/2}$, where K is any nonzero constant.

7. $\dfrac{dr}{dt} + 2tr = r \;\Rightarrow\; \dfrac{dr}{dt} = r - 2tr = r(1 - 2t) \;\Rightarrow\; \displaystyle\int \frac{dr}{r} = \int (1 - 2t)\,dt \;\Rightarrow\; \ln|r| = t - t^2 + C \;\Rightarrow$
$|r| = e^{t - t^2 + C} = ke^{t - t^2}$. Since $r(0) = 5$, $5 = ke^0 = k$. Thus, $r(t) = 5e^{t - t^2}$.

8. $(1 + \cos x)y' = (1 + e^{-y})\sin x \;\Rightarrow\; \dfrac{dy}{1 + e^{-y}} = \dfrac{\sin x\,dx}{1 + \cos x} \;\Rightarrow\; \displaystyle\int \frac{dy}{1 + 1/e^y} = \int \frac{\sin x\,dx}{1 + \cos x} \;\Rightarrow$
$\displaystyle\int \frac{e^y\,dy}{1 + e^y} = \int \frac{\sin x\,dx}{1 + \cos x} \;\Rightarrow\; \ln|1 + e^y| = -\ln|1 + \cos x| + C \;\Rightarrow\; \ln(1 + e^y) = -\ln(1 + \cos x) + C \;\Rightarrow$
$1 + e^y = e^{-\ln(1 + \cos x)} \cdot e^C \;\Rightarrow\; e^y = ke^{-\ln(1 + \cos x)} - 1 \;\Rightarrow\; y = \ln[ke^{-\ln(1 + \cos x)} - 1]$. Since $y(0) = 0$,
$0 = \ln[ke^{-\ln 2} - 1] \;\Rightarrow\; e^0 = k(\tfrac{1}{2}) - 1 \;\Rightarrow\; k = 4$. Thus, $y(x) = \ln[4e^{-\ln(1 + \cos x)} - 1]$. An equavalent form is
$y(x) = \ln \dfrac{3 - \cos x}{1 + \cos x}$.

9. $\dfrac{d}{dx}(y) = \dfrac{d}{dx}(ke^x) \;\Rightarrow\; y' = ke^x = y$, so the orthogonal trajectories must have $y' = -\dfrac{1}{y} \;\Rightarrow\; \dfrac{dy}{dx} = -\dfrac{1}{y} \;\Rightarrow$
$y\,dy = -dx \;\Rightarrow\; \int y\,dy = -\int dx \;\Rightarrow\; \tfrac{1}{2}y^2 = -x + C \;\Rightarrow\; x = C - \tfrac{1}{2}y^2$, which are parabolas with a horizontal axis.

10. $\frac{d}{dx}(y) = \frac{d}{dx}(e^{kx}) \Rightarrow y' = ke^{kx} = ky = \frac{\ln y}{x} \cdot y$, so the orthogonal trajectories must have $y' = -\frac{x}{y \ln y} \Rightarrow$ $\frac{dy}{dx} = -\frac{x}{y \ln y} \Rightarrow y \ln y \, dy = -x \, dx \Rightarrow \int y \ln y \, dy = -\int x \, dx \Rightarrow \frac{1}{2}y^2 \ln y - \frac{1}{4}y^2$ [parts with $u = \ln y$, $dv = y\,dy$] $= -\frac{1}{2}x^2 + C_1 \Rightarrow 2y^2 \ln y - y^2 = C - 2x^2$.

11. (a) $y(t) = y(0)e^{kt} = 200e^{kt} \Rightarrow y(0.5) = 200e^{0.5k} = 360 \Rightarrow e^{0.5k} = 1.8 \Rightarrow 0.5k = \ln 1.8 \Rightarrow$ $k = 2 \ln 1.8 = \ln(1.8)^2 = \ln 3.24 \Rightarrow y(t) = 200e^{(\ln 3.24)t} = 200(3.24)^t$

(b) $y(4) = 200(3.24)^4 \approx 22{,}040$ bacteria

(c) $y'(t) = 200(3.24)^t \cdot \ln 3.24$, so $y'(4) = 200(3.24)^4 \cdot \ln 3.24 \approx 25{,}910$ bacteria per hour

(d) $200(3.24)^t = 10{,}000 \Rightarrow (3.24)^t = 50 \Rightarrow t \ln 3.24 = \ln 50 \Rightarrow t = \ln 50 / \ln 3.24 \approx 3.33$ hours

12. (a) If $y(t)$ is the mass remaining after t years, then $y(t) = y(0)e^{kt} = 100e^{kt}$. $y(5.24) = 100e^{5.24k} = \frac{1}{2} \cdot 100 \Rightarrow$ $e^{5.24k} = \frac{1}{2} \Rightarrow 5.24k = -\ln 2 \Rightarrow k = -\frac{1}{5.24} \ln 2 \Rightarrow y(t) = 100e^{-(\ln 2)t/5.24} = 100 \cdot 2^{-t/5.24}$. Thus, $y(20) = 100 \cdot 2^{-20/5.24} \approx 7.1$ mg.

(b) $100 \cdot 2^{-t/5.24} = 1 \Rightarrow 2^{-t/5.24} = \frac{1}{100} \Rightarrow -\frac{t}{5.24} \ln 2 = \ln \frac{1}{100} \Rightarrow t = 5.24 \frac{\ln 100}{\ln 2} \approx 34.8$ years

13. (a) $C'(t) = -kC(t) \Rightarrow C(t) = C(0)e^{-kt}$ by Theorem 10.4.2. But $C(0) = C_0$, so $C(t) = C_0 e^{-kt}$.

(b) $C(30) = \frac{1}{2}C_0$ since the concentration is reduced by half. Thus, $\frac{1}{2}C_0 = C_0 e^{-30k} \Rightarrow \ln \frac{1}{2} = -30k \Rightarrow$ $k = -\frac{1}{30} \ln \frac{1}{2} = \frac{1}{30} \ln 2$. Since 10% of the original concentration remains if 90% is eliminated, we want the value of t such that $C(t) = \frac{1}{10}C_0$. Therefore, $\frac{1}{10}C_0 = C_0 e^{-t(\ln 2)/30} \Rightarrow \ln 0.1 = -t(\ln 2)/30 \Rightarrow t = -\frac{30}{\ln 2} \ln 0.1 \approx 100$ h.

14. (a) Let $t = 0$ correspond to 1990 so that $P(t) = 5.28e^{kt}$ is a starting point for the model. When $t = 10$, $P = 6.07$. So $6.07 = 5.28e^{10k} \Rightarrow 10k = \ln\frac{6.07}{5.28} \Rightarrow k = \frac{1}{10}\ln\frac{6.07}{5.28} \approx 0.01394$. For the year 2020, $t = 30$, and $P(30) = 5.28e^{30k} \approx 8.02$ billion.

(b) $P = 10 \Rightarrow 5.28e^{kt} = 10 \Rightarrow \frac{10}{5.28} = e^{kt} \Rightarrow kt = \ln\frac{10}{5.28} \Rightarrow t = 10\dfrac{\ln\frac{10}{5.28}}{\ln\frac{6.07}{5.28}} \approx 45.8$ years; that is, in $1990 + 45 = 2035$.

(c) $P(t) = \dfrac{K}{1 + Ae^{-kt}} = \dfrac{100}{1 + Ae^{-kt}}$, where $A = \dfrac{100 - 5.28}{5.28} \approx 17.94$. Using $k = \dfrac{1}{10}\ln\dfrac{6.07}{5.28}$ from part (a), a model is $P(t) \approx \dfrac{100}{1 + 17.94e^{-0.01394t}}$ and $P(30) \approx 7.81$ billion, slightly lower than our estimate of 8.02 billion in part (a).

(d) $P = 10 \Rightarrow 1 + Ae^{-kt} = \frac{100}{10} \Rightarrow Ae^{-kt} = 9 \Rightarrow e^{-kt} = 9/A \Rightarrow -kt = \ln(9/A) \Rightarrow$ $t = -\dfrac{1}{k}\ln\dfrac{9}{A} \approx 49.47$ years (that is, in 2039), which is later than the prediction of 2035 in part (b).

15. (a) $\dfrac{dL}{dt} \propto L_\infty - L \Rightarrow \dfrac{dL}{dt} = k(L_\infty - L) \Rightarrow \displaystyle\int \frac{dL}{L_\infty - L} = \int k\,dt \Rightarrow -\ln|L_\infty - L| = kt + C \Rightarrow$ $\ln|L_\infty - L| = -kt - C \Rightarrow |L_\infty - L| = e^{-kt-C} \Rightarrow L_\infty - L = Ae^{-kt} \Rightarrow L = L_\infty - Ae^{-kt}$. At $t = 0$, $L = L(0) = L_\infty - A \Rightarrow A = L_\infty - L(0) \Rightarrow L(t) = L_\infty - [L_\infty - L(0)]\,e^{-kt}$.

(b) $L_\infty = 53$ cm, $L(0) = 10$ cm, and $k = 0.2 \Rightarrow L(t) = 53 - (53 - 10)e^{-0.2t} = 53 - 43e^{-0.2t}$.

16. (a) If $y = u - 20$, $u(0) = 80 \Rightarrow y(0) = 80 - 20 = 60$, and the initial-value problem is $dy/dt = ky$ with $y(0) = 60$. So the solution is $y(t) = 60e^{kt}$. Now $y(0.5) = 60e^{k(0.5)} = 60 - 20 \Rightarrow e^{0.5k} = \frac{40}{60} = \frac{2}{3} \Rightarrow k = 2\ln\frac{2}{3} = \ln\frac{4}{9}$, so $y(t) = 60e^{(\ln 4/9)t} = 60(\frac{4}{9})^t$. Thus, $y(1) = 60(\frac{4}{9})^1 = \frac{80}{3} = 26\frac{2}{3}$ °C and $u(1) = 46\frac{2}{3}$ °C.

(b) $u(t) = 40 \Rightarrow y(t) = 20$. $y(t) = 60(\frac{4}{9})^t = 20 \Rightarrow (\frac{4}{9})^t = \frac{1}{3} \Rightarrow t\ln\frac{4}{9} = \ln\frac{1}{3} \Rightarrow t = \dfrac{\ln\frac{1}{3}}{\ln\frac{4}{9}} \approx 1.35$ h or 81.3 min.

17. Let P represent the population and I the number of infected people.

The rate of spread dI/dt is jointly proportional to I and to $P - I$, so for some constant k, $dI/dt = kI(P - I)$ $\Rightarrow$

$I = \dfrac{I_0 P}{I_0 + (P - I_0)e^{-kPt}}$ [from the discussion of logistic growth in Section 7.5].

Now, measuring t in days, we substitute $t = 7$, $P = 5000$, $I_0 = 160$ and $I(7) = 1200$ to find k:

$$1200 = \frac{160 \cdot 5000}{160 + (5000 - 160)e^{-5000 \cdot 7 \cdot k}} \quad \Leftrightarrow \quad 3 = \frac{2000}{160 + 4840e^{-35{,}000k}} \quad \Leftrightarrow \quad 480 + 14{,}520e^{-35{,}000k} = 2000 \quad \Leftrightarrow$$

$$e^{-35{,}000k} = \frac{2000 - 480}{14{,}520} \quad \Leftrightarrow \quad -35{,}000k = \ln\frac{38}{363} \quad \Leftrightarrow \quad k = \frac{-1}{35{,}000}\ln\frac{38}{363} \approx 0.00006448.$$ Next, let

$I = 5000 \times 80\% = 4000$, and solve for t: $4000 = \dfrac{160 \cdot 5000}{160 + (5000 - 160)e^{-k \cdot 5000 \cdot t}} \quad \Leftrightarrow \quad 1 = \dfrac{200}{160 + 4840e^{-5000kt}} \quad \Leftrightarrow$

$$160 + 4840e^{-5000kt} = 200 \quad \Leftrightarrow \quad e^{-5000kt} = \frac{200 - 160}{4840} \quad \Leftrightarrow \quad -5000kt = \ln\frac{1}{121} \quad \Leftrightarrow$$

$t = \dfrac{-1}{5000k}\ln\dfrac{1}{121} = \dfrac{1}{\frac{1}{7}\ln\frac{38}{363}} \cdot \ln\dfrac{1}{121} = 7 \cdot \dfrac{\ln 121}{\ln\frac{363}{38}} \approx 14.875$. So it takes about 15 days for 80% of the population to be infected.

18. Denote the amount of salt in the tank (in kg) by y. $y(0) = 0$ since initially there is only water in the tank.

The rate at which y increases is equal to the rate at which salt flows into the tank minus the rate at which it flows out.

That rate is $\dfrac{dy}{dt} = 0.1\dfrac{\text{kg}}{\text{L}} \times 10\dfrac{\text{L}}{\text{min}} - \dfrac{y}{100}\dfrac{\text{kg}}{\text{L}} \times 10\dfrac{\text{L}}{\text{min}} = 1 - \dfrac{y}{10}\dfrac{\text{kg}}{\text{min}} \quad \Rightarrow \quad \displaystyle\int \frac{dy}{10 - y} = \int \frac{1}{10}\,dt \quad \Rightarrow$

$-\ln|10 - y| = \frac{1}{10}t + C \quad \Rightarrow \quad 10 - y = Ae^{-t/10}$. $y(0) = 0 \quad \Rightarrow \quad 10 = A \quad \Rightarrow \quad y = 10\left(1 - e^{-t/10}\right)$.

At $t = 6$ minutes, $y = 10\left(1 - e^{-6/10}\right) \approx 4.512$ kg.

19. $\dfrac{dh}{dt} = -\dfrac{R}{V}\left(\dfrac{h}{k + h}\right) \quad \Rightarrow \quad \displaystyle\int \frac{k + h}{h}\,dh = \int \left(-\frac{R}{V}\right)dt \quad \Rightarrow \quad \int \left(1 + \frac{k}{h}\right)dh = -\frac{R}{V}\int 1\,dt \quad \Rightarrow$

$h + k\ln h = -\dfrac{R}{V}t + C$. This equation gives a relationship between h and t, but it is not possible to isolate h and express it in terms of t.

20. $dx/dt = 0.4x - 0.002xy$, $dy/dt = -0.2y + 0.000008xy$

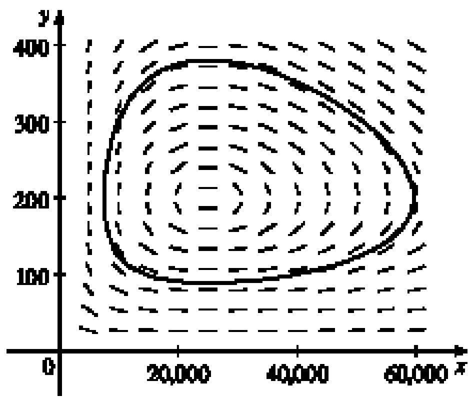
(a) The xy terms represent encounters between the birds and the insects. Since the y-population increases from these terms and the x-population decreases, we expect y to represent the birds and x the insects.

(b) x and y are constant $\Rightarrow$ $x' = 0$ and $y' = 0$ $\Rightarrow$

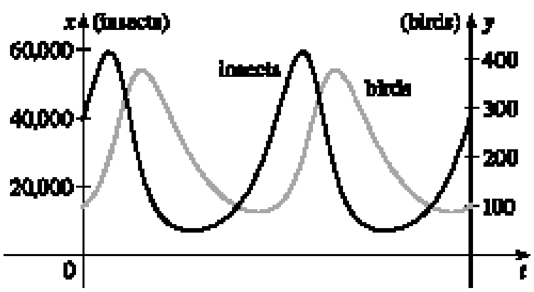
$$\left.\begin{cases} 0 = 0.4x - 0.002xy \\ 0 = -0.2y + 0.000008xy \end{cases}\right\} \Rightarrow \begin{cases} 0 = 0.4x(1 - 0.005y) \\ 0 = -0.2y(1 - 0.00004x) \end{cases} \Rightarrow y = 0 \text{ and } x = 0 \text{ (zero populations)}$$

or $y = \frac{1}{0.005} = 200$ and $x = \frac{1}{0.00004} = 25{,}000$. The non-trivial solution represents the population sizes needed so that there are no changes in either the number of birds or the number of insects.

(c) $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{-0.2y + 0.000008xy}{0.4x - 0.002xy}$

(d) 

At $(x, y) = (40{,}000, 100)$, $dx/dt = 8000 > 0$, so as t increases we are proceeding in a counterclockwise direction. The populations increase to approximately $(59{,}646, 200)$, at which point the insect population starts to decrease. The birds attain a maximum population of about 380 when the insect population is 25,000. The populations decrease to about $(7370, 200)$, at which point the insect population starts to increase. The birds attain a minimum population of about 88 when the insect population is 25,000, and then the cycle repeats.

(e) 

Both graphs have the same period and the bird population peaks about a quarter-cycle after the insect population.

21. (a) $dx/dt = 0.4x(1 - 0.000005x) - 0.002xy$, $dy/dt = -0.2y + 0.000008xy$. If $y = 0$, then $dx/dt = 0.4x(1 - 0.000005x)$, so $dx/dt = 0 \quad \Leftrightarrow \quad x = 0$ or $x = 200{,}000$, which shows that the insect population increases logistically with a carrying capacity of 200,000. Since $dx/dt > 0$ for $0 < x < 200{,}000$ and $dx/dt < 0$ for $x > 200{,}000$, we expect the insect population to stabilize at 200,000.

(b) x and y are constant $\Rightarrow$ $x' = 0$ and $y' = 0$ $\Rightarrow$

$$\left.\begin{cases} 0 = 0.4x(1 - 0.000005x) - 0.002xy \\ 0 = -0.2y + 0.000008xy \end{cases}\right\} \Rightarrow \begin{cases} 0 = 0.4x[(1 - 0.000005x) - 0.005y] \\ 0 = y(-0.2 + 0.000008x) \end{cases}$$

The second equation is true if $y = 0$ or $x = \frac{0.2}{0.000008} = 25{,}000$. If $y = 0$ in the first equation, then either $x = 0$ or $x = \frac{1}{0.000005} = 200{,}000$. If $x = 25{,}000$, then $0 = 0.4(25{,}000)[(1 - 0.000005 \cdot 25{,}000) - 0.005y] \Rightarrow$ $0 = 10{,}000[(1 - 0.125) - 0.005y] \Rightarrow 0 = 8750 - 50y \Rightarrow y = 175$.

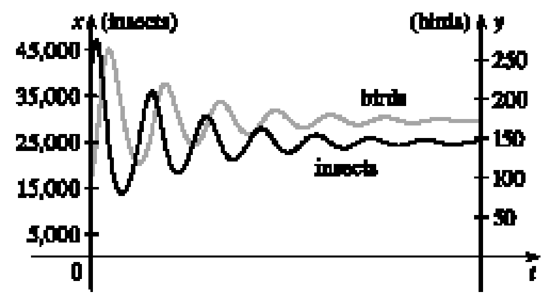
Case (i): $y = 0$, $x = 0$: Zero populations

Case (ii): $y = 0$, $x = 200{,}000$: In the absence of birds, the insect population is always 200,000.

Case (iii): $x = 25{,}000$, $y = 175$: The predator/prey interaction balances and the populations are stable.

(c) The populations of the birds and insects fluctuate around 175 and 25,000, respectively, and eventually stabilize at those values.

(d)



22. First note that, in this question, "weighs" is used in the informal sense, so what we really require is Barbara's mass m in kg as a function of t. Barbara's net intake of calories per day at time t (measured in days) is $c(t) = 1600 - 850 - 15m(t) = 750 - 15m(t)$, where $m(t)$ is her mass at time t. We are given that $m(0) = 60$ kg and $\frac{dm}{dt} = \frac{c(t)}{10{,}000}$, so $\frac{dm}{dt} = \frac{750 - 15m}{10{,}000} = \frac{150 - 3m}{2000} = \frac{-3(m - 50)}{2000}$ with $m(0) = 60$. From $\int \frac{dm}{m - 50} = \int \frac{-3\,dt}{2000}$, we get $\ln|m - 50| = -\frac{3}{2000}t + C$. Since $m(0) = 60$, $C = \ln 10$. Now $\ln \frac{|m - 50|}{10} = -\frac{3t}{2000}$, so $|m - 50| = 10e^{-3t/2000}$. The quantity $m - 50$ is continuous, initially positive, and the right-hand side is never zero. Thus, $m - 50$ is positive for all t, and $m(t) = 50 + 10e^{-3t/2000}$ kg. As $t \to \infty$, $m(t) \to 50$ kg. Thus, Barbara's mass gradually settles down to 50 kg.

1. True. Since $y^4 \geq 0$, $y' = -1 - y^4 < 0$ and the solutions are decreasing functions.

2. True. $y = \dfrac{\ln x}{x} \quad \Rightarrow \quad y' = \dfrac{1 - \ln x}{x^2}$.

LHS $= x^2 y' + xy = x^2 \cdot \dfrac{1 - \ln x}{x^2} + x \cdot \dfrac{\ln x}{x} = (1 - \ln x) + \ln x = 1 =$ RHS, so $y = \dfrac{\ln x}{x}$ is a solution of $x^2 y' + xy = 1$.

3. False. $x + y$ cannot be written in the form $g(x)f(y)$.

4. True. $y' = 3y - 2x + 6xy - 1 = 6xy - 2x + 3y - 1 = 2x(3y - 1) + 1(3y - 1) = (2x + 1)(3y - 1)$, so y' can be written in the form $g(x)f(y)$, and hence, is separable.

5. True. By comparing $\dfrac{dy}{dt} = 2y\left(1 - \dfrac{y}{5}\right)$ with the logistic differential equation (7.5.1), we see that the carrying capacity is 5; that is, $\lim\limits_{t \to \infty} y = 5$.