

# Math 1b

## Final Review - Integration

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# Resources for Review

- \* **Review Guides**

<http://www.courses.fas.harvard.edu/~math1b/exams/>

- \* **Exams and solutions from previous years**

<http://www.courses.fas.harvard.edu/~math1b/prevexams/>

- \* **Solutions to the Chapter Review Exercises**

<http://www.courses.fas.harvard.edu/~math1a/exams/>

# Exam Particulars

- \* Tuesday, May 23 at 2:15-5:15 PM in Geology Lecture Hall
- \* No calculators allowed
- \* All out-of-sequence exams must be approved by the Final Exams Office

# Techniques of Integration

- \* Integration by substitution
- \* Integration by parts
- \* Partial fractions
- \* Trigonometric integrals

- $\int \sin^3 x \, dx$

- $\int x e^{2x} \, dx$

- $\int \cos^2 x \, dx$

- $\int \frac{-x + 1}{x^2 + 3x + 2} \, dx$

# Numerical Integration

- \* Right and Left Sums
- \* The Midpoint Rule
- \* The Trapezoid Rule
- \* Simpson's Rule

# Applications of Integration

- \* Area between curves
- \* Volumes
- \* Density
- \* Arclength
- \* Average Value of a Function
- \* Work

# Volumes

- Find the volume of the region bounded by  $y = x^2$  and  $y = x$  when rotated about the line  $y = 2$ .
- The base of a solid is the region between the curve  $y = 2\sqrt{\sin x}$  on the interval  $[0, \pi]$  on the  $x$ -axis. The cross-sections perpendicular to the  $x$ -axis are squares with bases running from the  $x$ -axis to the curve. Find the volume of the solid.

# Arc Length

Find the arc length of the curve

$$y = \frac{1}{3}x^3 + \frac{1}{4x}$$

from  $x = 1$  to  $x = 3$ .

# Work

- A mountain climber is about to haul up a 150 ft length of hanging rope. If the rope weighs  $\frac{1}{4}$  lb/ft, how much work will be done in lifting the rope?
- A spherical tank with a diameter of 8 ft is filled with water to a depth of 6 ft. How much work is done in pumping all of the liquid to a height of 2 ft above the top of the tank? The density of the liquid is  $\rho = 40$  lb/ft<sup>3</sup>.

# Setting Up Riemann Sums

A coastal town is in the shape of a 7 by 2 mile rectangle, with one of the 7 mile sides bordering the coast. In this town, people want to live near the beach and the population density at a distance  $x$  from the coast is given by

$$\delta(x) = 4000 - 2000x$$

people per square mile.

- (a) Write a Riemann sum that approximates the total population of the town.
- (b) Write a definite integral that represents the total area of the town.
- (c) What is the population of the town?