Answers (and Partial Solutions) to Problems from the Review Sheet used in the Review Session (and not solved there)

6. Revolve the region R described about the y-axis.

   Slice horizontally - so each slice is a disk with thickness Δy.

   Volume of ith disk $\approx \pi(r_i)^2\Delta y$ where $r_i = x_i = y^{1/3}$

   Volume of ith disk $\approx \pi(y_i)^{2/3}\Delta y$

   Total volume $= \int_0^8 \pi(y)^{2/3}\,dy = \pi(3/5)y^{5/3}|_0^8 = \pi(3/5)8^{5/3}$

   Now integrate to H and set the value of the integral equal to half of what we just got.

   $\int_0^H \pi(y)^{2/3}\,dy = \pi(3/5)y^{5/3}|_0^H = \pi(3/5)H^{5/3}$

   Set $\pi(3/5)H^{5/3} = \frac{1}{2}\pi(3/5)8^{5/3}$ So $H^{5/3} = \frac{1}{2}8^{5/3}$ or $H = 8/2^{3/5}$.

   This makes sense since $2^{2/5}$ is approximately 1.31955 - quite a bit more than half the total height. (Notice that the parfait cup is narrowest at the bottom.)

9. I'll set zero to be on the ground and the positive y direction to be up. I'll calculate the work to move the basket and then add in the work to move the rope. I'll let g be the gravitational constant 9.8 rmm/sec^2.

   Work = Force$\cdot$ Distance $\text{Work}_{\text{basket}} = 15\text{kg}\cdot g\text{m/sec}^2\cdot 5 = 75g$ Joules

   To this answer we add $\text{Work}_{\text{rope}}$.

   The rope's weight changes as it is pulled up. Chop the distance the rope moves through. [0,5].

   The work done to move the rope through the ith distance interval $\approx$ weight $\cdot\,\Delta y$

   $\approx 1(5 - y_i)g\Delta y$

   $\text{Work}_{\text{rope}} = \int_0^5 (5-y)g\,dy = \ldots g25/2.$ Joules

   Total work $= (75 + 12.5)g = 87.5g$ Joules

11. Note- the fountain is 2 feet tall, not 1 foot tall.

    (a) Slice from y = 0 to y = 1.8 parallel to the base. Use a uniform partition. The weight of the ith slice is approximately $\rho(y)\pi r^2\Delta y$ where
    r_i = the x value on the $y = \ln x$ curve.
    In other words, $r_i = e^{y_i}$.
    The weight of the ith slice is approximately $\rho(y)\pi e^{2y_i}\Delta y$
    Summing and taking the limit as the number of slices grows without bound gives $\int_0^{1.8} \rho(y)\pi e^{2y}\,dy$

    (b) Let y be the distance from the bottom of the fountain. Again, slice parallel to the base.We' ll need to multiply the weight of a slice by the distance the slice must travel.
    The work required to lift the ith slice over the rim is approximately $\rho(y)pi{r_i}^2\Delta y(2?y_i)$
    where r = the x value on the $y = \ln x$ curve.
    The work required to lift the ith slice over the rim is approximately $\rho(y)pie^{2y_i}\Delta y(2?y_i)$
    The total work necessary is given by $\int_0^{1.8} \rho(y)\pi e^{2y}(2-y)\,dy$

12. First you have to figure out how far the moose has travelled in two hours: (Note: velocity is in miles per hour and all axes for the position curve are labeled in miles.)

    Distance moose travels in 2 hours. $\int_0^2 v(t)\,dt = \ldots 2t^2 + (1/2)t|_0^2 = 9$. So the moose travelled 9 miles.

    Now we have to figure out where the dear moose is. We want the length along the curve to be equal to 9.

    Arc Length $= \int_0^X \sqrt{1 + (dy/dx)^2}\,dx$.

    $dy/dx = x\sqrt{x^2+2}$

    so $1 + (dy/dx)^2 = 1 + x^2(x^2+2) = x^4 + 2x^2 + 1 = (x^2+1)^2$ (Careful to use the Chain Rule.)

    Thus, Arc Length $= \int_0^X \sqrt{(x^2+1)^2}\,dx = \int_0^X x^2 + 1\,dx = 1/3X^3 + X$. Set this equal to 9. This is not easy to solve. (But we've been lucky with our numbers so far - and this is an arc length problem - so you can't have everything.) To estimate a solution to $1/3X^3 + X = 9$ you can use a calculator or Newton's method. (Or even successive guessing and a calculator.) The answer is just under 2.68.