Integration Review

1. Let R be the region from $x = 3$ to $x = 20$ bounded above by the graph of $y = \frac{10}{x}$ and below by the $x-$axis. Suppose we rotate the region R about each of the lines given below. Write an integral that gives the volume of revolution.

   (a) Rotate about the $x-$axis.

   (b) Rotate about the vertical line $y = -2$.

2. Now let R be the region bounded above by the graph of $y = \frac{10}{x}$, on the left by $x = 3$ and below by the $x-$axis. Is the area under the curve finite? Rotate the area about the $x-$axis. You get an object known as "Gabriel's Horn". Does Gabriel's horn have finite volume? Curious . . .

3. Two of your young classmates are having some trouble with improper integrals. They are discussing $\int_0^\infty f(x)\,dx$ where f is defined, positive, and bounded on [0,1) and $\lim_{x\to\infty} f(x) = 0$. Margaret believes that $\int_0^\infty f(x)\,dx$ ought to diverge. She reasons that if f is positive then then the accumulated area keeps increasing, even if only little bit, so we can't get anything other than infinity since $\int_0^b f(x)\,dx$ increases with b and we need to let b go to infinity.

   Amani, on the other hand, is convinced that if $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ ought to converge. After all, he reasons, the rate at which area is accumulating is going to zero. That should be enough assure convergence. Margaret and Amani ask you for assistance. Explain very clearly the errors each of them are making. There is really not enough information given about $\int_0^\infty f(x)\,dx$ to draw any conclusion. Illustrate this by providing two integrals of this form (satisfying the conditions given) one of which converges and one of which diverges.

4. Why is the integral $\displaystyle\int_0^\infty \frac{dx}{x\ln x}$ improper?

5. For each of the following integrals, determine whether or not the integral converges.

   (a) $\displaystyle\int_2^\infty \frac{dx}{x\ln x}$

   (b) $\displaystyle\int_2^\infty \frac{dx}{x(\ln x)^2}$

   (c) $\displaystyle\int_2^\infty \frac{100e^{-x}dx}{x(\ln x)^4}$

   (d) $\displaystyle\int_2^\infty \frac{\sqrt{1+\sin^2 x}\,dx}{x}$

6. A parfait cup is formed by revolving the curve $y = x^3, 0 \le x \le 2$ about the $y-$ axis. The parfait cut is filled to the brim with hot chocolate. If you plan to drink exactly half the hot chocolate in the cup, what height should the liquid be when you stop drinking? *(Think about your problem solving strategy.)*

7. A chocolate truffle is a wonderfully decadent chocolate concoction. Truffles tend to be spherical or hemispherical.

   (a) Consider a truffle made by dipping a round hazelnut into various chocolates, building up a delicious chocolate delicacy. The number of calories per cubic millimeter varies with x, the distance from the center of the hazelnut. If $\rho(x)$ gives the calories per cubic millimeter at a distance x millimeters from the center, write an integral that gives the number of calories in a truffle of radius R.

   (b) Another truffle is made in a hemispherical mold of radius R. (The mold looks like a tiny hemispherical bowl.) Different layers of chocolate are poured into the mold, one at a time, and allowed to set. The number of calories per cubic millimeter varies with x, the distance from the top of the mold. The caloric density is given by $\delta(x)$ calories per cubic millimeter. Write an integral that gives the number of calories in this hemispherical truffle.

8. (Dubious food problem continued:) The density of flavoring syrup in a slush cup varies with the distance from the bottom of the container. Suppose $f(x)$ gives the number of ounces of syrup per cubic inch, where x is the distance from the bottom of the cup. Write an integral giving the total amount of syrup in the cup if the cup is a cone with a height of 8 inches and a radius of 6 inches.

9. Filled with the desire to be away from the troubles on the ground nearby, you have climbed a tree which is 5 meters high. So that you will be able to stay there for a while, you have arranged a basket containing lunch and some reading materials, weighing 15 kg in all. Before climbing up the tree you have attached a sturdy chain to the basket the chain weighs 1 kg per meter. How much work do you do pulling the picnic basket up to you via the chain?

10. Consider a conical tank with radius 10 meters and height 4 meters which is completely filled with water. The top of the tank lies at ground level, so you can't empty it by siphoning the water out. Calculate the work done in pumping out the water out into a waiting truck 1 meter about the top of the tank. (The mass density of water is 1000 kilograms per cubic meter, and the acceleration due to gravity is 9.8 meters per second squared.)

11. A fountain in a Newport mansion has a shape given by revolving region bounded by the x and y axis, $y = 2$ and the curve $y = \ln x$ about the y axis. The fountain is 1 foot tall; the radius of its base is 1 foot. The fountain sits on the front lawn. After a hurricane it contains murky silty water. The weight of the silt-laden water varies with the height and is given by $\rho(y)$ lbs per cubic foot where y is the distance from the lawn. If the fountain is filled with murky water to a height of 1.8 feet,

    (a) what is the weight of the stuff in the fountain?

    (b) how much work is required to empty the fountain by emptying the murky water over the top rim of the fountain?

12. A moose is walking along a path. Its velocity is given by $v(t) = 4t + 1/2$. The path it is traveling on is well approximated by the curve $y = (1/3)(x^2 + 2)^{3/2}$ and at $t = 0$, the moose is at the point where $x = 0$, walking in the direction of increasing x values. What will its $x-$coordinate be two hours later?

13. A 2 foot long storage unit in a boat has varying widths. Measurements are taken every 6 inches and are given (in feet) below.

3	5	6	10	8

    (a) Use the trapezoidal rule to approximate the storage area.

    (b) Use Simpson's rule to approximate the storage area.

    (c) What is the average width? (Can you get a better average than (3+5+6+10 + 8)/5 = 6.4)

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