

Math 1b. Calculus, Series, and Differential Equations Review Guide for Midterm II Spring 2006

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Midterm Details

The first midterm will primarily cover Chapter 8 (omit Section 8.8), but you will also be responsible for any previous material covered in the course. The midterm exam will be on Tuesday, April, 18 at 7-9 PM in Science B. There will also be a course-wide review session on Friday, April 14 at 3-5 PM in Science D. We plan to videotape the review session, and you should be able to access the video by clicking on Lecture Videos at the course website.

Studying and Reviewing

- You can find copies of old midterms clicking on Previous Exams at the course website. Solutions to these exams will be posted on the course website no later than Friday, April 14. Course assistants will conduct exam preparation workshops this week in section, and you may pick up an advance copy of the solutions to the previous exams by attending any CA section. A list of CA section times can be found at <http://www.courses.fas.harvard.edu/~math1b/sections/>.
- You should also try working some of the problems in the review sections of Chapter 8 (pp. 631-633). We will post solutions to these problems on the course website no later than Friday, April 14. You may pick up an advance copy of the solutions by attending any CA section.

- Be sure to take advantage of the TF office hours, CA sections, and the MQC in Science 309.

Topics for Midterm II

- To understand and be able to the notion of a sequence of numbers. (Section 8.1)
- To understand and be able to apply the notions of iteratively defined and recursively defined sequences. (Section 8.1)
- To understand and be able to apply the definitions of convergence and divergence of a sequence.¹ (Section 8.1)
- To understand a be able to apply the various properties of sequences. (Section 8.1)
- To understand and be able to apply the fact that a bounded monotonic sequence converges. (Section 8.1)
- To understand the definition of a series. (Section 8.2)
- To understand and be able to apply the basic properties of series. (Section 8.2)
- To understand and be able to apply the Divergence Test. (Section 8.2)
- To understand and be able to apply geometric series. (Section 8.2)
- To understand and be able to apply the integral test to find the convergence of a series. (Section 8.3)
- To understand and be able to the remainder of a series using the Integral Test. (Section 8.3)
- To understand and be able to tell if a p -series converges. (Section 8.3)
- To understand and be able to apply the Comparison Test to determine the convergence or divergence of a series.² (Section 8.3)
- To understand and be able to apply the definition of an alternating series. (Section 8.4)
- To understand and be able to apply the concepts of conditional and absolute convergence. (Section 8.4)

¹We do not cover the formal definition of a convergent sequence.

²We will not require that you know the Limit Comparison Test. You can, however, use the Limit Comparison Test on any problem unless the problem asks you to use a specific test or specifically forbids the use of the Limit Comparison Test.

- To understand and be able to apply the ratio test to determine the convergence or divergence of a series. (Section 8.4)
- To understand and be able to apply the idea of a power series. (Section 8.5)
- To be able to determine the radius and interval of convergence of a power series. (Section 8.5)
- To understand and be able to represent functions as power series. (Section 8.6)
- To be able to differentiate and integrate power series to obtain new ways to represent functions as power series. (Section 8.6)
- To understand and be able to compute Taylor and Maclaurin series. (Section 8.7)
- To understand and be able to find the Taylor polynomial of a function. (Section 8.7)
- To understand the convergence of a Taylor series. (Section 8.7)
- To understand and be able to make computations using Taylor's Inequality and Taylor's Remainder Theorem. (Section 8.7)
- To understand how Taylor polynomials can be used to approximate functions. (Section 8.9)

Strategies for Determining the Convergence or Divergence of a Series

1. If the series is of the form $\sum 1/n^p$, then it is a p -series. The series converges for $p > 1$ and diverges for $p \leq 1$.
2. If the series has the form $\sum ar^n$, then it is a geometric series and converges for $|r| < 1$ and diverges for $|r| \geq 1$.
3. If the series is similar to a p -series or a geometric series, consider the Comparison Test.
4. If $\lim_{n \rightarrow \infty} a_n \neq 0$, then the series diverges.
5. If the series is of the form $\sum (-1)^{n+1} a_n$, consider applying the Alternating Series Test. You can also test for absolute convergence.
6. If the series involves products, factorials, or constants raised to the n th power, consider the Ratio Test.
7. If $a_n = f(n)$ and the integral $\int_1^\infty f(x) dx$ is easily evaluated, the Integral Test may be useful assuming the hypothesis of the test are satisfied.
8. Is the series a telescopic series? If so, convergence or divergence can be determined by computing the limit of the partial sums of the series.

Some Useful Power Series³

Function	Series	Interval of Convergence
e^x	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	$(-\infty, \infty)$
$\sin x$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$	$(-\infty, \infty)$
$\cos x$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$	$(-\infty, \infty)$
$\frac{1}{1-x}$	$1 + x + x^2 + x^3 + \dots$	$(-1, 1)$
$\frac{1}{1+x}$	$1 - x + x^2 - x^3 + \dots$	$(-1, 1)$
$\frac{1}{1+x^2}$	$1 - x^2 + x^4 - x^6 + \dots$	$(-1, 1)$
$\arctan x$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$[-1, 1]$
$\ln(x+1)$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$	$(-1, 1)$

Taylor's Remainder Theorem and Taylor's Inequality⁴

- If $|f^{(n+1)}| \leq M$ between a and x , then the remainder of the Taylor series satisfies the inequality

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}.$$

- If f has continuous derivatives of orders $1, 2, \dots, n+1$ between a and x , then

$$f(x) = T_n(x) + \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1}$$

where c lies between a and x .

Notice that the Mean Value Theorem is a special case of Taylor's Remainder Theorem.

³You should know the power series for e^x , $\sin x$, $\cos x$, and $1/(1-x)$.

⁴This material is different than what is presented in Stewart.