Math 1b
Midterm II Review

T. Judson
April 14, 2006

Resources for Review

* **Midterm II Review Guide**
  http://www.courses.fas.harvard.edu/~math1b/exams/Midterm2Sp05review.pdf

* **Exams and solutions from previous years** http://www.courses.fas.harvard.edu/~math1b/prevexams/

* **Solutions to the Chapter 8 Review Exercises** http://www.courses.fas.harvard.edu/~math1a/exams/

Exam Particulars

* Tuesday, April 18 at 7-9 PM in Science Center B

* No calculators allowed

* All out-of-sequence exams must be approved by the course head

* No make-up exams

What to Expect

* Approximately nine questions (some with several parts)

* The emphasis will be on material from Chapter 8 (omit Section 8.8)

* Refer to the Midterm II Review Guide for details

Sequences

A sequence is a list of numbers, $a_1, a_2, a_3, \ldots$. More formally, we can think of a sequence as a function

$$a : N \to R,$$

where $a(n) = a_n$. Give a example of a sequence that is:

1. convergent but not monotone.

2. monotone but not convergent.

3. bounded but not monotone.

4. monotone decreasing and unbounded.

5. monotone increasing and convergent.

6. unbounded but not monotone.

7. bounded and monotone but not convergent

Geometric Series

Let

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + \cdots .$$

- If $|r| < 1$, then $\displaystyle\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$.

- If $|r| \geq 1$, the series diverges.

1. Express $0.131313\ldots$ as a ratio of two integers.

2. A ball is dropped from a height of 8 ft. Each time the ball bounces, it comes back up to one-half of its previous height. What is the total distance that the ball travels?

Series

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2 + n + 2}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{n^2 + 2^n}$

3. $\displaystyle\sum_{n=2}^{\infty} \frac{5^n}{n!}$

4. $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{3^k + 4^k}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\ln(3 + n^2)}$

6. $\displaystyle\sum_{n=1}^{\infty} n e^{-n^2}$

1. If the series is of the form $\sum 1/n^p$, then it is a p-series. The series converges for $p > 1$ and diverges for $p \leq 1$.

2. If the series has the form $\sum ar^n$, then it is a geometric series and converges for $|r| < 1$ and diverges for $|r| \geq 1$.

3. If the series is similar to a p-series or a geometric series, consider the Comparison Test.

4. If $\lim_{n \to \infty} a_n \neq 0$, then the series diverges.

5. If the series is of the form $\sum (-1)^{n+1} a_n$, consider applying the Alternating Series Test. You can also test for absolute convergence.

6. If the series involves products, factorials, or constants raised to the nth power, consider the Ratio Test.

7. If $a_n = f(n)$ and the integral $\int_1^\infty f(x)\, dx$ is easily evaluated, the Integral Test may be useful assuming the hypothesis of the test are satisfied.

8. Is the series a telescopic series? If so, convergence or divergence can be determined by computing the limit of the partial sums of the series.

Alternating Series

Let

$$\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + \cdots$$

satisfy the following conditions.

1. $a_1 \geq a_2 \geq a_3 \geq \cdots$

2. $\lim_{n \to \infty} a_n = 0$

Then the series converges and

$$|R_n| = |S - S_n| \leq a_{n+1}.$$

Power Series Representations

Find the interval of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{(x-1)^k}{k4^k}$$

Important Power Series

Function	Series	Interval of Convergence
e^x	$1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$	$(-\infty, \infty)$
$\sin x$	$x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \cdots$	$(-\infty, \infty)$
$\cos x$	$1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \cdots$	$(-\infty, \infty)$
$\dfrac{1}{1-x}$	$1 + x + x^2 + x^3 + \cdots$	$(-1, 1)$

Taylor Series

Find the Taylor series centered at $x = 0$ for

$$\sinh x = \frac{e^x - e^{-x}}{2}.$$

Taylor Polynomials

- Find the second-degree Taylor polynomial for $f(x) = \sqrt{x}$ at the number $a = 100$.

- Approximate $\sqrt{101}$.

- Estimate the error between the approximation and $\sqrt{101}$.

Taylor Polynomials and Error

If $|f^{(n+1)}| \leq M$ between a and x, then the remainder of the Taylor series satisfies the inequality

$$|f(x) - T_n(x)| = |R_n(x)| \leq \frac{M}{(n+1)!}|x - a|^{n+1}.$$