Name:_____

Math 1b Midterm I
Tuesday, March 14, 2006

Please circle your section:

Rina Anno	Li-Sheng Tseng	Robert Strain
10–11 MWF	10–11 MWF	11–12 MWF

Thomas Judson	Robin Gottlieb	Robin Gottlieb
11–12 MWF	10–11:30 TTh	11:30–1 TTh

Problem Number	Possible Points	Score
1	6	
2	11	
3	15	
4	11	
5	11	
6	11	
7	11	
8	11	
9	13	
Total	100	

Directions—Please Read Carefully! You have two hours to take this exam. Make sure to use correct mathematical notation. To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit. Please be sure to write neatly—illegible answers will receive little or no credit. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers. *Calculators are not allowed.* **Good Luck!!!**

1. (6 points) Which of the following gives the area bounded on the left by the y-axis, on the right by the curve $y = 3\arcsin x$ and above by $y = 3\pi/2$? Please circle ***ALL*** correct answers. No written justification is required.

   (a) $\int_0^1 (3\pi/2 - 3\arcsin x)\, dx$

   (b) $\int_0^{3\pi/2} (3\pi/2 - 3\arcsin x)\, dx$

   (c) $\int_0^{3\pi/2} \sin(\frac{y}{3})\, dy$

   (d) $\int_0^1 \sin\frac{y}{3}\, dy$

   (e) $\frac{1}{3}\int_0^{3\pi/2} \sin y\, dy$

2. (11 points)

   (a) Let $f(x) = 3x^2 - 2x + k$. Find k such that the average value of f on the interval $[-1, 2]$ is 3.

   (b) Suppose that f is a continuous function. If the average value of f over the interval $[-3, 1]$ is 2 and the average value of f over the interval $[1, 7]$ is 5, what is the average value of the function over the interval $[-3, 7]$? ***Hint***: The answer is not $7/2$.

3. (15 points) Consider the region bounded curves

$$\begin{aligned} y &= x^2 - 1 \\ y &= -x + 1 \end{aligned}$$

(a) Sketch the area bounded by these curves.

(b) Calculate the area between both of these curves.

(c) Find the volume of the solid of revolution obtained by rotating the region enclosed by the two curves around the horizontal line $y = -1$.

4. (11 points) The soot produced by a garbage incinerator spreads out in a circular pattern. The depth of soot that is currently on the ground is $H(r) = e^{-2r}/10$ millimeters deep, where r is the number of kilometers from the incinerator. What is the total volume of soot that has been deposited within a 5 km radius of the incinerator? Remember that 1000 mm is one meter and 1000 m is one kilometer.

5. (11 points) A snail crawls along the curve $y = \sqrt{x^3}$ at a speed of 3 ft/hr. How long does it take the snail to crawl from the point $(1, 1)$ to $(4, 8)$, where the x and y-coordinates are given in feet.

6. (11 points) A cylindrical gasoline tank with radius 4 ft and length 15 ft is buried under a service station. The top of the tank is 10 ft underground, and its flat ends are perpendicular to the ground's surface. Find a definite integral that will tells us the total amount of work needed to pump all of the gasoline in the tank to a nozzle that is 3 ft above the ground. (Gasoline weighs $\rho = 42$ lb/ft^3.) ***You do not need to evaluate the integral***.

7. (11 points) The base of a solid object is the region bounded by $y = 1/x$, $y = 0$, $x = 1$, and $x = 4$. Every cross-section of the solid taken perpendicular to the x-axis is a square. What is the volume of the object?

8. (11 points) Hoping to estimate the volume of wood in a 20-meter log, Ranger Smith uses a tape measure to gauge the log's girth (circumference) at 5 meter intervals, starting from the large end of the log. Here are Ranger Smith's results.

Measuring a Natural Log					
distance from the end (m)	0	5	10	15	20
circumference (m)	4.0	3.6	3.2	2.6	2.0

(a) Let x be the distance in meters from the large end of the log and $c(x)$ be the girth of the log at x. Write a definite integral that approximates the volume of the log.

(b) Write out an arithmetic expression that uses the Trapezoid Rule to estimate the volume of the log for $n = 4$. ***You do not need to evaluate the expression.***

(c) Write out an arithmetic expression that uses the Simpson's Rule to estimate the volume of the log for $n = 4$. ***You do not need to evaluate the expression.***

9. (13 points)

(a) Find $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$.

(b) Determine whether

$$\int_1^\infty \frac{2}{1+x^5}\,dx$$

converges. Explain your reasoning completely.

(c) Determine whether

$$\int_{-\infty}^\infty \sin 2x\,dx$$

converges or diverges.