Math 1b Midterm II
Tuesday, April 18, 2006

Please circle your section:

Rina Anno	Li-Sheng Tseng	Robert Strain
10–11 MWF	10–11 MWF	11–12 MWF

Thomas Judson	Robin Gottlieb	Robin Gottlieb
11–12 MWF	10–11:30 TTh	11:30–1 TTh

Problem Number	Possible Points	Score
1	6	
2	20	
3	20	
4	6	
5	6	
6	4	
7	10	
8	20	
9	8	
Total	100	

Directions—Please Read Carefully! You have two hours to take this exam. Make sure to use correct mathematical notation. To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit. Please be sure to write neatly—illegible answers will receive little or no credit. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers. *Calculators are not allowed.* **Good Luck!!!**

1. (6 points)   Suppose that the power series $\sum_{n=0}^{\infty} a_n(x+2)^n$ converges if $x = -7$ and diverges if $x = 7$. Decide which of the following series must converge, must diverge, or may either converge or diverge (inconclusive). Circle your answer. *You do not need to justify your answers.*

   (a) If $x = -8$, the power series   *Converges*   *Diverges*   *Inconclusive*

   (b) If $x = 1$, the power series   *Converges*   *Diverges*   *Inconclusive*

   (c) If $x = 3$, the power series   *Converges*   *Diverges*   *Inconclusive*

   (d) If $x = -11$, the power series   *Converges*   *Diverges*   *Inconclusive*

   (e) If $x = 5$, the power series   *Converges*   *Diverges*   *Inconclusive*

   (f) If $x = -5$, the power series   *Converges*   *Diverges*   *Inconclusive*

2. (20 points)   Determine whether each series below converges absolutely, converges conditionally, or diverges. Be careful to justify each of your answers by explicitly referring to the test that you used and explaining how you used it.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4 + n + 1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{1}{3^n} \right)^2$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^3}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{2^n}{n!(\ln n)^3}$

(e) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

3. (20 points)   Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = -2$ is given by

$$T_3(x) = 2 - \frac{3}{8}(x+2)^2 - \frac{1}{12}(x+2)^3.$$

(a) Find $f(-2)$, $f'(-2)$, and $f''(-2)$.

(b) Determine whether f has a local minimum, a local maximum, or neither at $x = -2$. Justify your answer.

(c) Use $T_3(x)$ to find an approximation for $f(0)$.

(d) The fourth derivative of f satisfies the inequality

$$\left|f^{(4)}(x)\right| \leq \frac{1}{4}$$

for all x in the closed interval $[-2, 0]$. Find an error bound on the approximation for $f(0)$ that you found in part (c).

4. (6 points)   Suppose we know that $\sum_{n=1}^{\infty} a_n$ converges to 0.8. We are given *no* other information about the infinite series. For each of the following statements circle

- *True* if the statement *must* be true,
- *False* if the statement *must* be false, and
- *Inconclusive* if the statement could be either true or false.

You do not need to justify your answers.

(a) $\lim\limits_{n\to\infty} a_n = 0.8$ *True*   *False*   *Inconclusive*

(b) $\lim\limits_{n\to\infty} a_n = 0$ *True*   *False*   *Inconclusive*

(c) $\lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} > 1$ *True*   *False*   *Inconclusive*

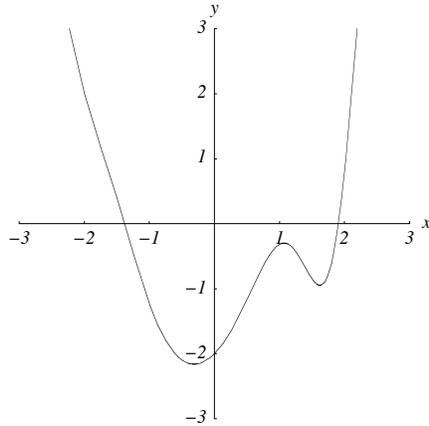(d) $a_{n+1} < a_n$ for all n *True*   *False*   *Inconclusive*

(e) $\lim\limits_{n\to\infty} \dfrac{1}{|a_n|} = \infty$ *True*   *False*   *Inconclusive*

(f) $\lim\limits_{n\to\infty} S_n = 0.8$,   *True*   *False*   *Inconclusive*
     where $S_n = a_1 + a_2 + \cdots + a_n$.

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5. (6 points)   The graph of $y = f(x)$ is given below. Assume that f is infinitely differentiable everywhere. The Taylor series for $f(x)$ about $x = 0$ is given by

$$f(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots .$$

(a) Determine whether c_0 is positive, negative, or zero. Explain your reasoning.

(b) Determine whether c_1 is positive, negative, or zero. Explain your reasoning.

(c) Determine whether c_2 is positive, negative, or zero. Explain your reasoning.

6. (4 points)   Consider the following power series.

(a) $x^2 - \dfrac{x^6}{3!} + \dfrac{x^{10}}{5!} - \cdots$

(b) $1 - \dfrac{x^4}{2!} + \dfrac{x^8}{4!} - \cdots$

(c) $\dfrac{x^2}{2!} - \dfrac{x^4}{4!} + \dfrac{x^6}{6!} - \cdots$

(d) $x - x^3 + x^5 - \cdots$

(e) $x^2 - \dfrac{x^4}{2!} + \dfrac{x^6}{4!} - \cdots$

Fill in the letter of the series that corresponds to the given function. *You do not need to justify your answer.*

_____ $1 - \cos x$

_____ $x^2 \cos x$

_____ $\sin(x^2)$

_____ $\dfrac{x}{1+x^2}$

7. (10 points)   Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x+5)^n}{n^2 3^n}.$$

If the interval of convergence is finite, make sure that you determine the convergence at each endpoint and justify your conclusions.

8. (20 points)

(a) Write down a power series expansion for e^{-x^3}.

(b) Write down a power series expansion for $\int e^{-x^3}\, dx$ and determine its radius of convergence.

(c) Use your answer in part (b) to find a series for $\int_0^{1/2} e^{-x^3}\, dx$.

(d) If you approximate the definite integral $\int_0^{1/2} e^{-x^3}\, dx$ by taking the partial sum consisting of the first four nonzero terms of the series that you obtained in part (c), what is the maximum error for your approximation.

9. (8 points)   Assume that the square $ABCD$ below has sides of *length one* and that E, F, G, and H are the midpoints of the sides.
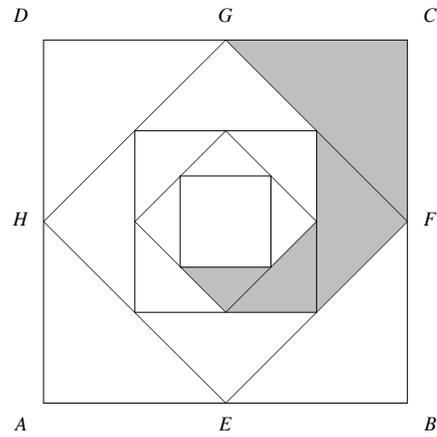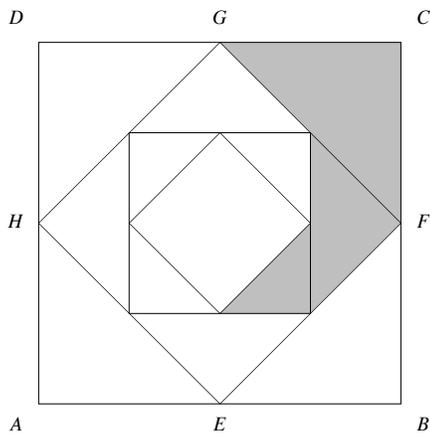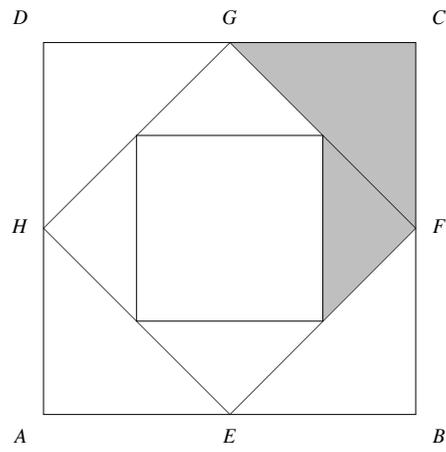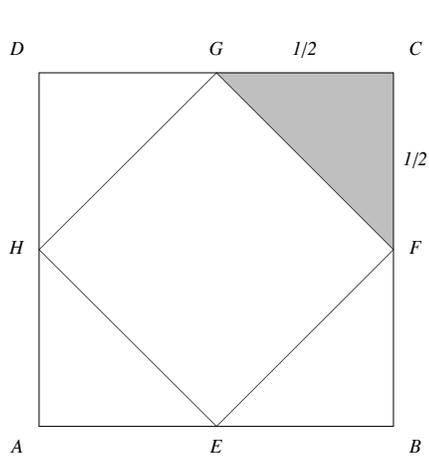
(a) If the indicated pattern is continued indefinitely, write an infinite series that will give the area of the shaded region?

(b) What is the area of the shaded region if the indicated pattern is continued indefinitely? That is, what is the sum of the series that you found in part (a)?