

Math 1b. Calculus, Series, and Differential Equations Review Guide for the Final Exam Spring 2006

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Final Exam Details

The final exam is comprehensive and will cover the entire course: integration, series and sequences, and differential equations. The final exam will be on Tuesday, May 23 at 2:15-5:15 in Geo Lecture Hall. There will also be three course-wide review sessions.

- Review session on differential equations: Thursday, May 11 at 2-4 PM in Science C
- Review session on integration: Monday, May 15 at 2-4 PM in Science C
- Review session on series and sequences: Wednesday, May 17 at 2-4 PM in Science C

We plan to videotape the review session, and you should be able to access the video by clicking on Lecture Videos at the course website.

Studying and Reviewing

- You can find copies of old exams clicking on Previous Exams at the course website

<http://www.courses.fas.harvard.edu/~math1b/prevexams/>

- You should also try working some of the problems in the review sections of Chapters 5–8. Solutions will be posted on the exam page.
- Be sure to take advantage of the TF office hours, CA sections, and the MQC in Science 309. Office hours are listed at

<http://www.courses.fas.harvard.edu/~math1b/exams/>

Topics for the Final Exam

For a list of topics on integration and series and sequences, see the review guides for the first and second midterms at

<http://www.courses.fas.harvard.edu/~math1b/exams/>

We will list the topics for differential equations below.

- To understand how differential equations can be used to model problems from the natural sciences, engineering, economics, and the social sciences (§7.1 in Stewart, §31.1 in Gottlieb).
- To understand what it means to be the solution of a differential equation (§7.1 in Stewart, §31.2 in Gottlieb).
- To understand the idea of a slope field (§7.1 in Stewart, §31.2 in Gottlieb).
- To understand and be able to apply the existence and uniqueness theorem (weak form) for differential equations (§7.1 in Stewart, §31.2 in Gottlieb).
- To be able to solve differential equations of the form $dy/dt = g(t)$ (§7.1 in Stewart, §31.2 in Gottlieb).
- To be able to solve differential equations of the form $dy/dt = ky$ (§7.1 in Stewart, §31.2 in Gottlieb).
- To understand that autonomous equations of the form $dy/dt = f(y)$ are time independent (§7.1 in Stewart, §31.2 in Gottlieb).
- To understand and be able to find equilibrium solutions for autonomous differential equations (§7.2 in Stewart, §31.3 in Gottlieb).
- To understand and be able to determine the stability of equilibrium solutions for autonomous differential equations (§7.2 in Stewart, §31.3 in Gottlieb).
- To understand and be able to draw the phase line for an autonomous differential equation and be able to use this information to sketch solutions of equations of the form $dy/dt = f(y)$ (§7.2 in Stewart, §31.3 in Gottlieb).
- To understand and be able to solve first-order separable differential equations (§7.3–5 in Stewart, §31.4 in Gottlieb).
- To be able model situations requiring first-order separable differential equations, such as mixing problems (§7.3–5 in Stewart, §31.4 in Gottlieb).
- To be able to solve first-order linear differential equations of the form

$$y' + p(t)y = q(t).$$

Handouts on first-order linear differential equations available at

http://www.courses.fas.harvard.edu/~math1b/handouts/linear_first_order.pdf <http://www.cour>

- To be able to model applications using first-order linear differential equations. Handouts on first-order linear differential equations available at http://www.courses.fas.harvard.edu/~math1b/handouts/linear_first_order.pdf <http://www.cour>
- To understand and be able to set up systems of first-order differential equations (§7.6 in Stewart, §31.5 in Gottlieb).
- To be understand and be able to analyze first-order systems by examining the phase plane (§7.6 in Stewart, §31.5 in Gottlieb).
- To understand and to be able to apply phase plane analysis to

$$\begin{aligned}\frac{dx}{dt} &= f(x, y), \\ \frac{dy}{dt} &= g(x, y).\end{aligned}$$

(§7.6 in Stewart, §31.5 in Gottlieb)

- To understand that the system

$$\begin{aligned}\frac{dx}{dt} &= f(x, y), \\ \frac{dy}{dt} &= g(x, y),\end{aligned}$$

is completely predictive. If you choose a starting point in the xy -plane, then there is exactly one solution that starts at your chosen point (§7.6 in Stewart, §31.5 in Gottlieb).

- To understand how use the equation $mx'' + bx' + kx = 0$ to model a harmonic oscillator (§31.6 in Gottlieb).
- To be able to solve equations of the form $ax'' + bx' + cx = 0$, when the roots of the characteristic equations are real and distinct (§31.6 in Gottlieb).
- To be able to solve equations of the form $ax'' + bx' + cx = 0$, when the roots of the characteristic equations are repeated and real or complex (§31.6 in Gottlieb).