Answers to Problems Given out at the Diff. Eq'ns Review (but not completed there)

2. a) $\boxed{\frac{dB}{dt} = 200 - .01B}$ —— $\boxed{B(0) = 5000}$

b) $\frac{dB}{dt} = 0$ at $200 = .01B$

$\boxed{B = 20{,}000}$

$\int \frac{dB}{200 - .01B} = \int dt$ or use shortcut

c) $\frac{dB}{dt} = -.01\left(\frac{200}{-.01} + B\right)$

$= -.01(-20{,}000 + B)$

$u = -20{,}000 + B$

$\frac{du}{dt} = -.01u$

$u = C_1 e^{-.01t}$

$\boxed{B = -15{,}000 e^{-.01t} + 20{,}000}$

$10{,}\cancel{000} = -15{,}\cancel{000} e^{-.01t} + 20{,}\cancel{000}$

$-10 = -15e^{-.01t}$

$\frac{2}{3} = e^{-.01t}$

$\ln\left(\frac{2}{3}\right) = -.01t$

$\frac{\ln\left(\frac{2}{3}\right)}{-.01} = \boxed{t = -100\ln(2/3)}$

3. $r^2 - 2r + 5 = 0$

$r = \frac{+2 \pm \sqrt{4 - 20}}{2} = 1 \pm \frac{\sqrt{16}i}{2} = 1 \pm 2i$

a) $\boxed{x(t) = e^t\left[C_1\cos 2t + C_2\sin 2t\right]}$

b) $r^2 - 5r + 6 = 0$

$(r-3)(r-2) = 0$

$x(t) = C_1e^{3t} + C_2e^{2t}$

$3C_1 + 2C_2 = 8$

$4 = C_1 + C_2$

$8 = 3C_1 + 2C_2$

$8 = 2C_1 + 2C_2$

$8 = 3C_1 + 2C_2$

$0 = C_1 \quad 0$

$C_1 = 0 \quad C_2 = 4$

$\boxed{x(t) = 4e^{2t}}$

c) $b = 0$ period $= 2 \Rightarrow \beta = \pi$

$\frac{\sqrt{4c}}{2} = \pi \qquad \sqrt{c} = \pi \Rightarrow c = \pi^2$

4 a (i) $\boxed{y = C_1e^t + C_2e^{2t}}$

(ii) Char. eqn: $(r-1)(r-2) = 0$

$r^2 - 3r + 2 = 0 \Rightarrow$ diff. eqn: $y'' - 3y' + 2y = 0$

$\boxed{b = -3} \; \boxed{c = 2}$

b) (i) $y'' + 3y' + 2y = 0 \Rightarrow r^2 + 3r + 2 = 0$

$(r+2)(r+1) = 0 \quad r = -2, r = -1$

$y = C_1e^{-2t} + C_2e^{-t}$ $\boxed{\text{Yes}}$

$\lim_{t\to\infty} C_1e^{-2t} + C_2e^{-t} = 0 + 0 = 0$

4b(i) (ii) $\boxed{\text{No}}$ - it doesn't have infinitely many zeros.

ii) $r^2 + r + 4 = 0$

$$r = \frac{-1 \pm \sqrt{1-16}}{2} = -\frac{1}{2} \pm \frac{\sqrt{15}\,i}{2} \qquad y = e^{-\frac{1}{2}t}\underbrace{\left[C_1 \cos \tfrac{\sqrt{15}}{2}t + C_2 \sin \tfrac{\sqrt{15}}{2}t\right]}$$

$\boxed{\lim_{t\to\infty} y = 0}$ ($e^{-\frac{1}{2}t} \to 0$ and this is bounded, - let's say $-M \le [C_1 \sim\sim\sim] \le M$

use the squeeze thm)

ii) $\boxed{\text{yes}}$ - there are infinitely many zeros $\left[C_1 \cos \frac{\sqrt{15}}{2}t + C_2 \sin \frac{\sqrt{15}}{2}t\right]$ is periodic

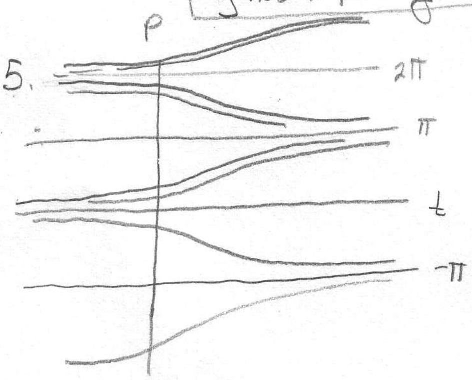
& equal to zero infinitely many times

iii) $r^2 - 2r + 3 = 0$

$$r = \frac{2 \pm \sqrt{4-12}}{2} = 1 \pm \sqrt{2}\,i \quad : y = e^t\left[C_1 \cos \sqrt{2}t + C_2 \sin \sqrt{2}t\right]$$

$\boxed{\lim_{t\to\infty} y \text{ does not exist.}}$

$\boxed{y \text{ has infinitely many zeros.}}$

5. 

b) If $P(0) = 6$ then we're in the $P \in [\pi, 2\pi]$ strip

$\boxed{\text{as } t \to \infty \quad P(t) \to \pi}$

c) If $P(0) = \pi$ $\boxed{\lim_{t\to\infty} P(t) = \pi}$ (π is an equilibrium soln)

d) $\frac{d^2P}{dt^2} = \frac{d}{dt}[\sin P] = \cos P\left(\frac{dP}{dt}\right) = \boxed{\cos P \sin P} = \boxed{\frac{d^2P}{dt^2}}$

7. System A: graph # $\boxed{7}$

System B: graph # $\boxed{3}$

System C: graph # $\boxed{2}$ (you can look at $\frac{dy}{dx}$ to distinguish between graphs 2 and 6)