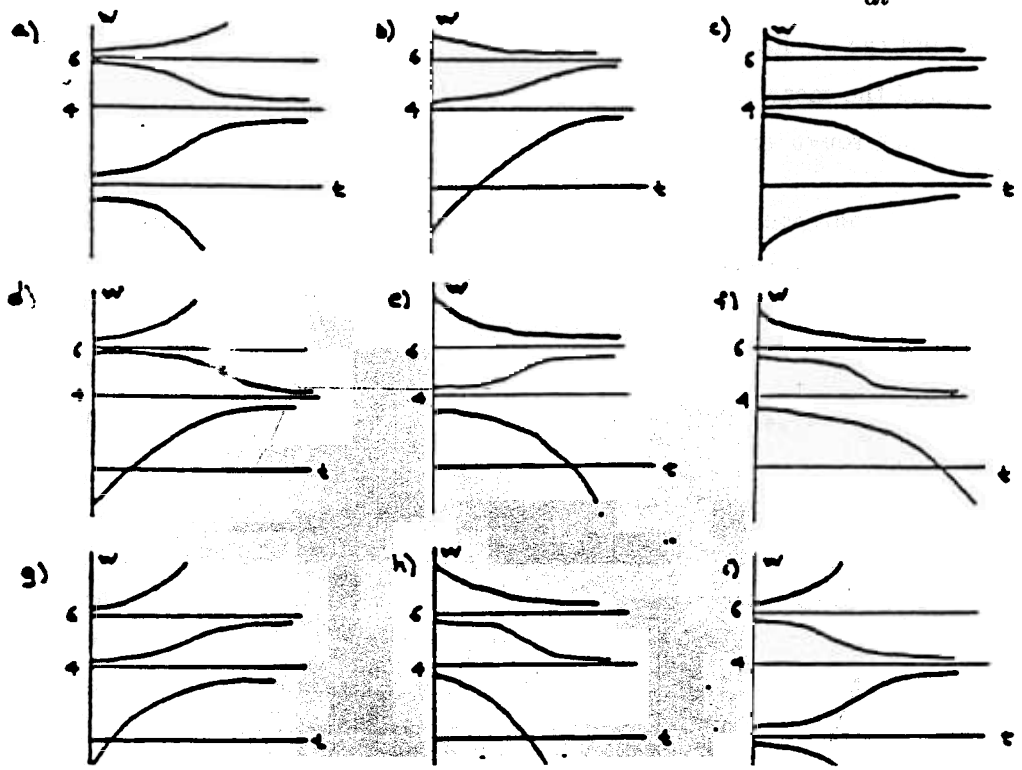
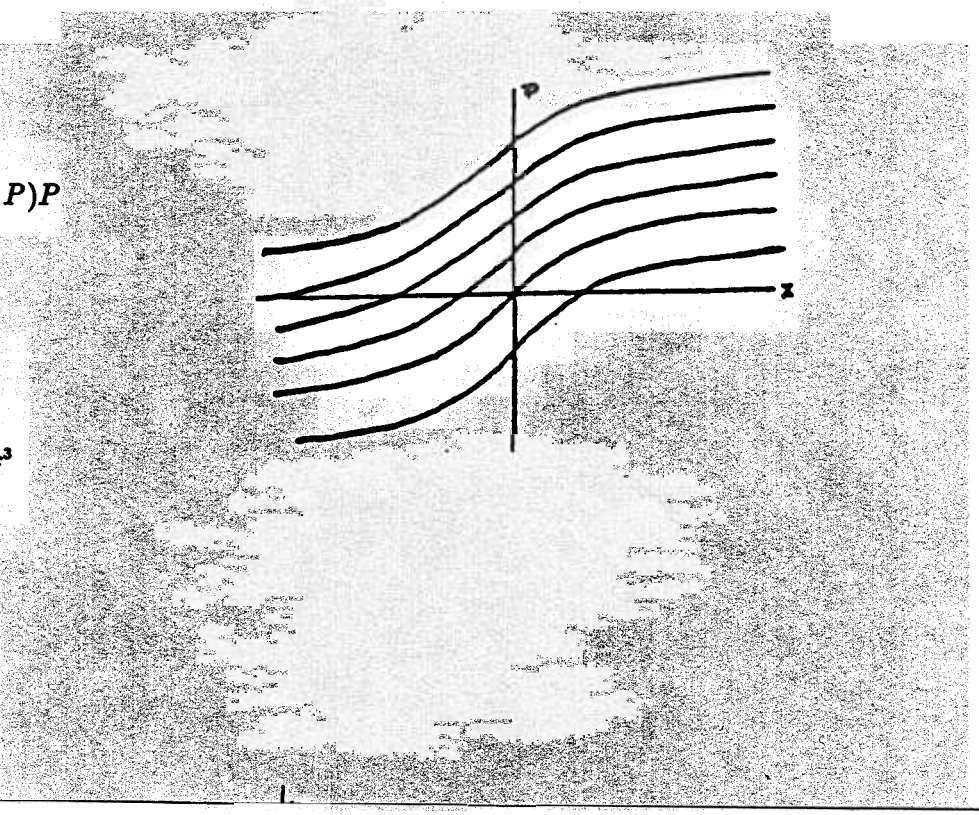
Review Problems for Differential Equations: Take Two

1. Which of the following is a graph of solutions to the differential equation $\frac{dw}{dt} = (w-6)(w-4)$?



2. Which one of the listed differential equations has solutions looking like those graphed below?



(a) $\frac{dP}{x} = -P^3$

(b) $\frac{P}{dx} = P$

(c) $\frac{dP}{dx} = (1-P)P$

(d) $\frac{dP}{dx} = x$

(e) $\frac{dP}{dx} = e^{-x^2}$

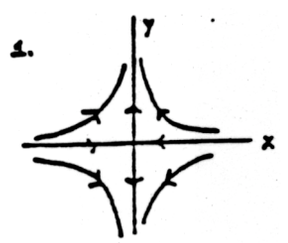
(f) $\frac{dP}{dx} = 3x\,e^{x^3}$

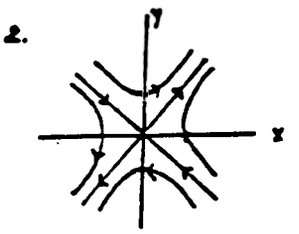
(a) $\dfrac{dx}{dt} = -2$ $\quad\dfrac{dy}{dt} = 4x$

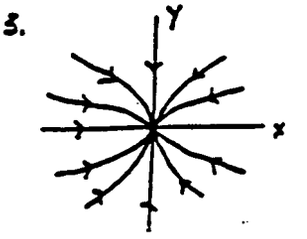
(b) $\dfrac{dx}{dt} = +3y$ $\quad\dfrac{dy}{dt} = -3x$

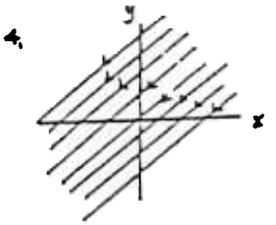
(c) $\dfrac{dx}{dt} = +10x$ $\quad\dfrac{dy}{dt} = +10y$

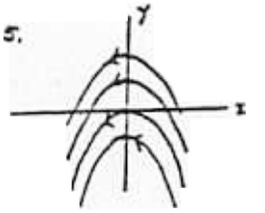
Match each of the pairs of differential equations with the correct figure below.

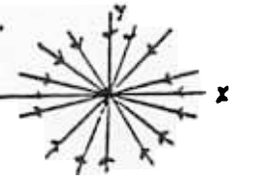


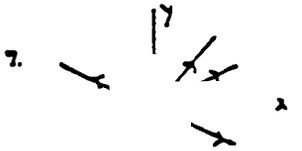


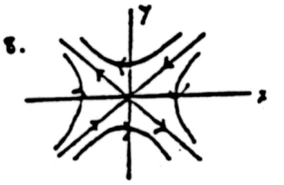


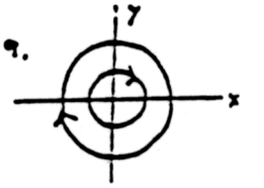


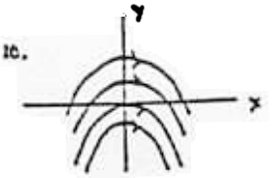


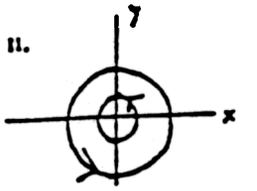


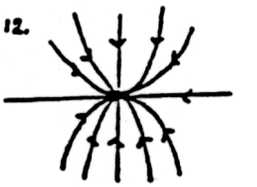












Answer: graph #_____ corresponds to system (a).
graph #_____ corresponds to system (b).
graph #_____ corresponds to system (c).

4. A porous material dries outdoors at a rate that is proportional to the moisture content. Set up a differential equation whose solution is $y = f(t)$, the amount of water at time t in a towel on a clothesline. Find and sketch the solution.

5. A savings account earns 6% annual interest compounded continuously. Withdrawals are made continuously at a rate of \$12,000 per year to fund a "continuous annuity."

(a) What is the smallest initial amount in the account that will fund such an annuity forever?

(b) What initial amount will fund such an annuity for exactly twenty years (at which time the savings account balance will be zero)?

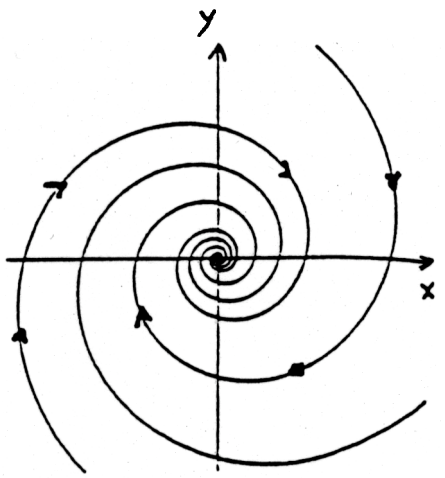
6. Which system of equations would roughly have the given phase plane diagram?

(a) $\frac{dx}{dt} = -10x + y$ $\quad \frac{dy}{dt} = x - 10y$

(b) $\frac{dx}{dt} = x - 10y$ $\quad \frac{dy}{dt} = 10x - y$

(c) $\frac{dx}{dt} = -x + 10y$ $\quad \frac{dy}{dt} = -10x - y$

(d) $\frac{dx}{dt} = x - 10y$ $\quad \frac{dy}{dt} = -10x - y$



7. Consider the Hakosalo residence in Oulu, Finland. Assume that heat is lost from the house only through windows and that the rate of change of temperature is proportional to the difference of temperature outside and inside. The constant of proportionality is $\frac{1}{29 \text{ hours}}$. Assume that it is 10°F outside constantly. On a Thursday at noon the temperature inside the house was 65°F and the heat was turned off until 5:00 pm.

(a) Write a differential equation which reflects the rate of change of the temperature in the house between 12:00 noon and 5:00 pm.

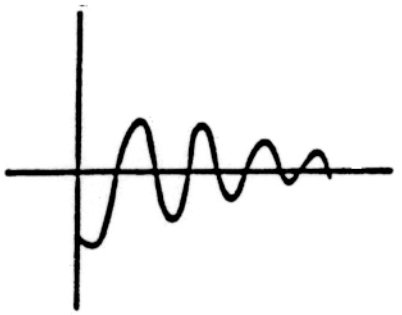
(b) Find the temperature in the house at 5:00 pm. (You may do this analytically or by using your calculator to get a rough estimate.)

(c) At 5:00 pm the heat is turned on. The heater generates an amount of energy that would raise the inside temperature by 2°F per hour if there were no heat loss. Write a differential equation that reflects what happens to the inside temperature after the heat is turned on.

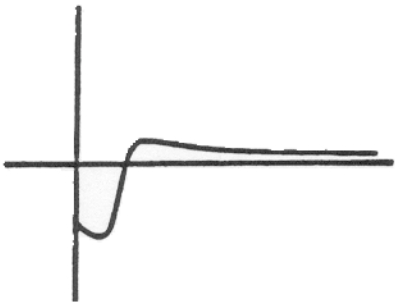
6. Which of the following is graph of the solution

$+x'+$ if $x(0)$, $x'(0)$

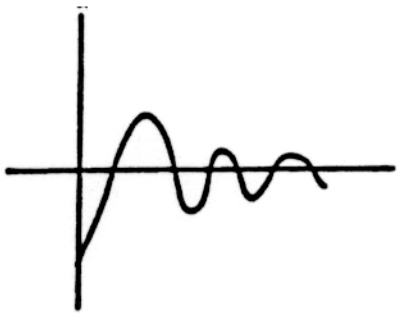
a)



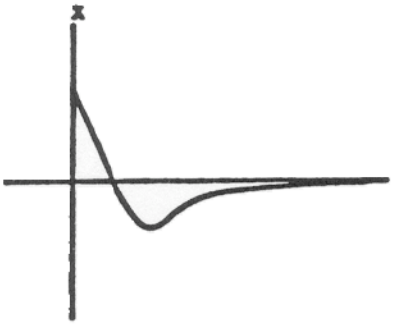
b)



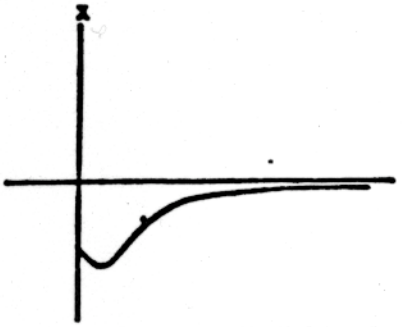
c)



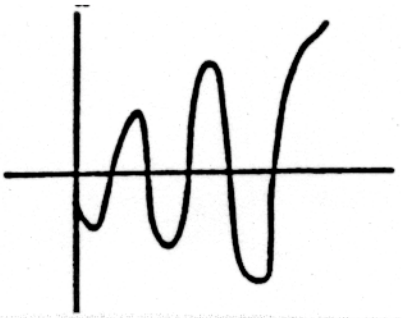
d)



e)

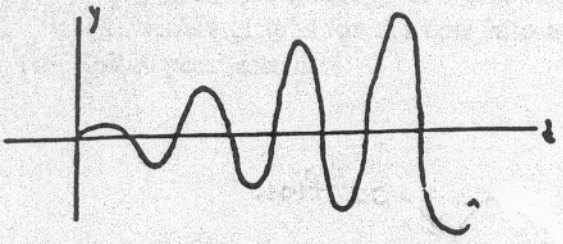


f)

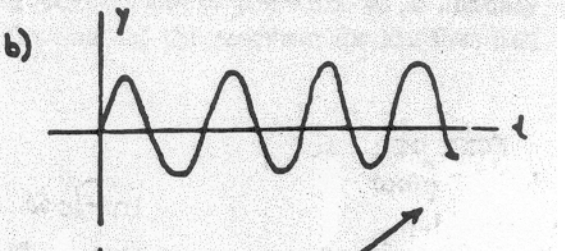


9. Which of the following is the graph of the solution to $\frac{d^2y}{dt^2} - \frac{dy}{dt} + y = 0$ and with the initial condition $y(0) = 0$, $y'(0) > 0$?

a)



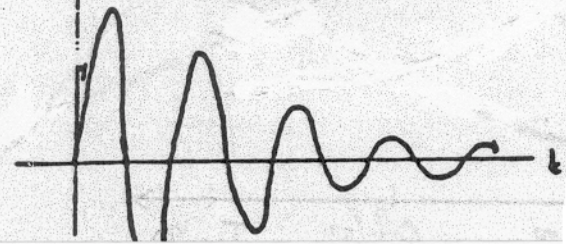
b)



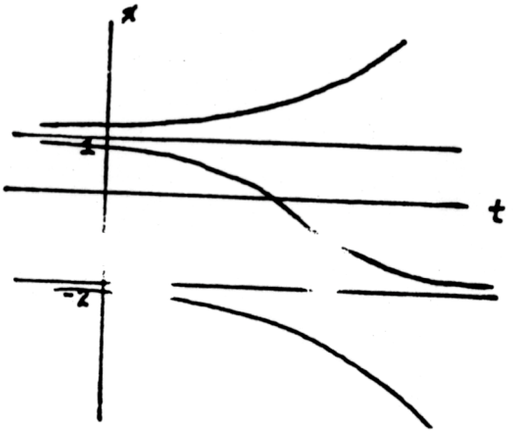
c)

d)

e)



10. Below is the graph of several particular solutions to a differential equation.



Which one of the following differential equations could have the solutions pictured above? Answer here: _____.

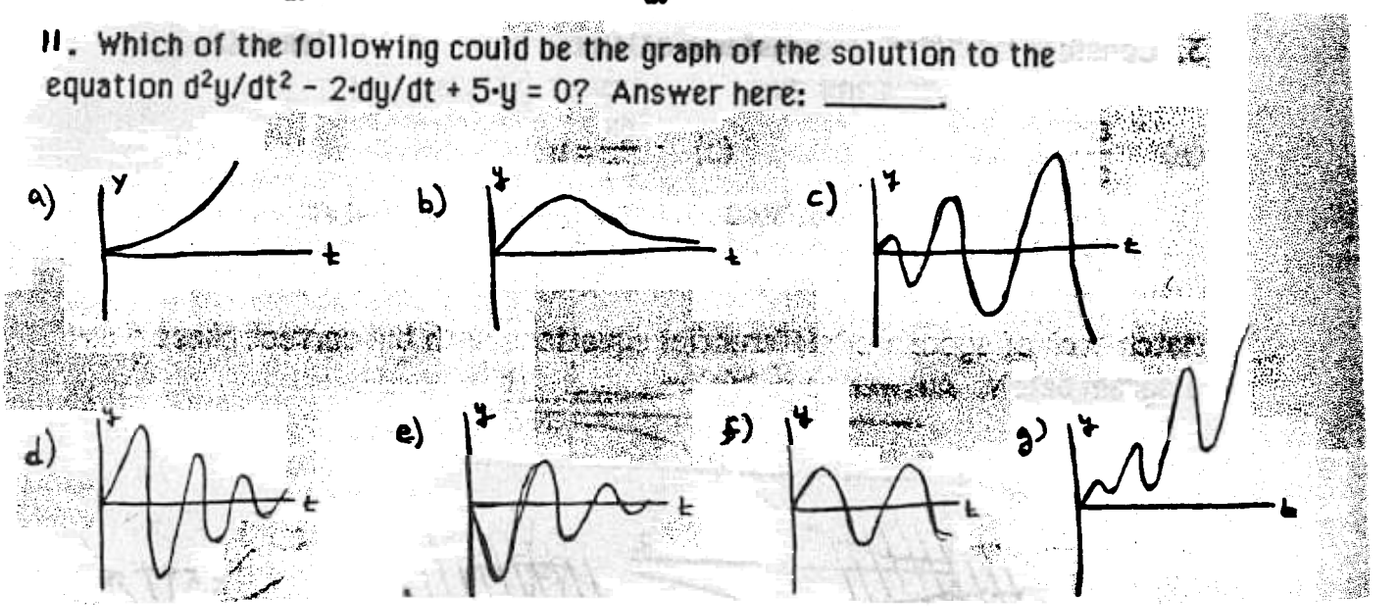
(a) $\frac{dx}{dt} = -(t-1)(t+2)$ (e) $\frac{dx}{dt} = x(x-1)(x+2)$

(b) $\frac{dx}{dt} = (t-1)(t+2)$ (f) $\frac{dx}{dt} = (x-1)(x+2)$

(c) $\frac{dx}{dt} = (t-1)^2(t+2)$ (g) $\frac{dx}{dt} = (x-1)^2(x+2)$

(d) $\frac{dx}{dt} = (t-1)(t+2)^2$ (h) $\frac{dx}{dt} = (x-1)(x+2)^2$

11. Which of the following could be the graph of the solution to the equation $d^2y/dt^2 - 2 \cdot dy/dt + 5 \cdot y = 0$? Answer here: _____



12. Which <u>one</u> of the following differential equations has a <u>stable</u> equilibrium at $w = 10$?

(a) $\frac{dw}{dt} = w - 10$ (c) $\frac{dw}{dt} = (w-10)^2$

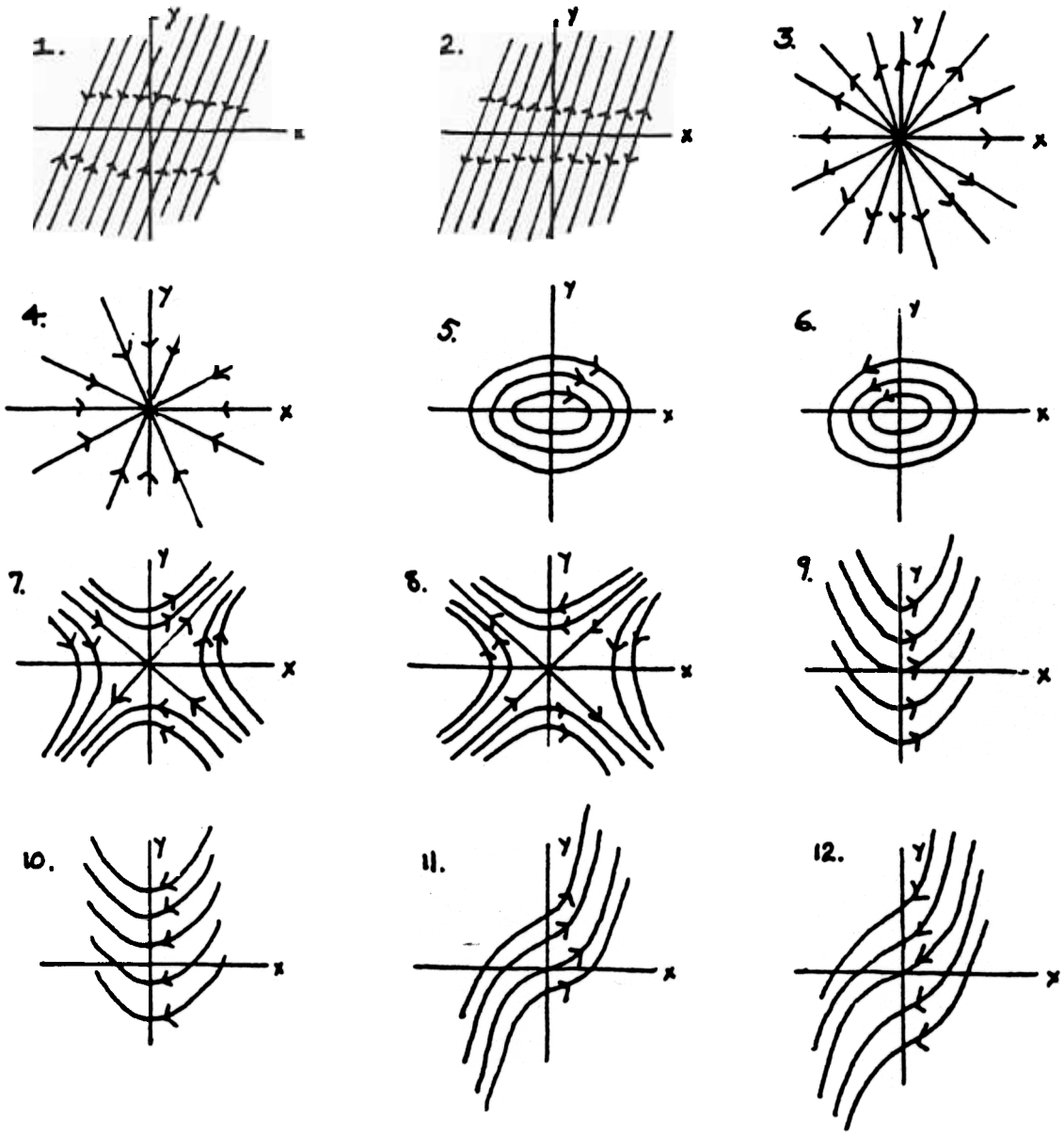
(b) $\frac{dw}{dt} = 10 - w$ (d) $\frac{dw}{dt} = -(w-10)^2$

13. Consider the following systems of differential equations:

(a) $\frac{dx}{dt} = y$
$\frac{dy}{dt} = -2x$

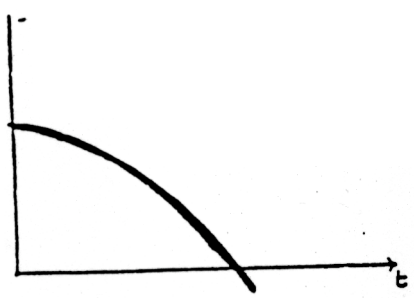
(b) $\frac{dx}{dt} = y$
$\frac{dy}{dt} = 3y$

Match each of system of differential equations with the correct phase plane diagram below. Answer here: (a) ↔ _____ (b) ↔ _____

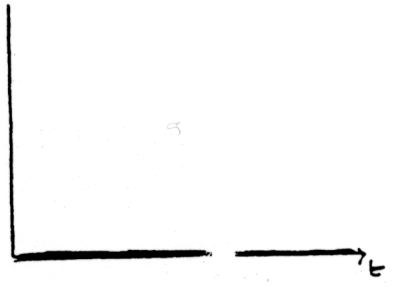


14. Each graph below is a solution to at least one of the differential equations. Match them up in such a way that each graph is used only once. No work need be shown.

A)



B)



C)

D)

(a) $\frac{d^2y}{dt^2} = 2$ has solution graph_____

(b) $\frac{dy}{dt} = y - 2$ has solution graph_____

(c) $\frac{dy}{dt} = y(2 - y)$ has solution graph_____

(d) $\frac{dy}{dt} = -yt$ has solution graph_____